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Abstracts

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Abstracts of Reports

On Finite-Dimensional Resolutions

Sergei Ageev

Belarus State University, BELARUS, ageev@hsu.by

Dranishnikov's resolution, a special type of a \((k - 1)\)-soft map between the \(k\)-dimensional Menger compactum and the Hilbert cube \(Q\), is an important technique of geometric topology revealing the wide analogy between \(Q\)-manifold and Menger manifold theories. On the other hand, by its properties, the Dranishnikov's resolution is a finite-dimensional analogue of the trivial \(Q\)-fibration over the Hilbert cube with the exception of being \(k\)-soft. It is clear that the more properties of Dranishnikov's resolution will be found the more convenient instrument it will become. The other bridge between the infinite dimensional and finite dimensional topologies (in fact, between Nöbeling and Hilbert \((l_2\) manifold theories) -- Chigogidze's resolution \(c_k : \nu^k \to l_2\), possesses properties which are every bit as remarkable as those of Dranishnikov's resolution. The opinion to consider \(c_k\) as a finite-dimensional analogue of the trivial \(l_2\)-fibration over the Hilbert space is justified to a greater degree than even in the case of Dranishnikov's resolution. We intend to focus our attention on the intimate interrelation between Dranishnikov's and Chigogidze's resolutions and to make a definite advance toward the investigation of these finite-dimensional resolutions.
On CSO-Compact Space

Jalal Hatem Hussein Albayati

Baghdad University, IRAQ, jalalintuch@yahoo.com

The aim of the talk is to introduce and study the concept of CSO-compact space via the notion of simply-open sets as well as to investigate their relationship to some well known classes of topological spaces and give some of their properties.

On the Telgarsky Conjecture Concerning Products of Paracompact Spaces

K. Alster

University of Cardinal Stefan Wyszyński and Institute of Mathematics of the Polish Academy of Sciences, POLAND, kalster@impan.gov.pl

The aim of the talk is to present some results concerning the Telgarsky conjecture which says that the product $X \times Y$ is paracompact for every paracompact space $Y$ if and only if the first player in the $G(\text{DC}, X)$ game, introduced by R. Telgarsky, has a winning strategy.

The Coarse Classification of Locally Finite and Abelian Groups

Taras Banakh, Ihor Zarichnyi and Jose Higes

Ivan Franko National University of Lviv, UKRAINE, tbanakh@yahoo.com

A multi-valued map $\Phi: X \to Y$ between two metric spaces is called bornologous if for each $\varepsilon$ there is $\delta$ such that for each subset $A \subseteq X$ of $\text{diam}(A) < \varepsilon$ the image $\Phi(A) = \bigcup_{a \in A} \Phi(a)$ has $\text{diam} \Phi(A) < \delta$. Two metric spaces $X, Y$ are called coarsely equivalent (or else asymorphic) if there is a multi-valued map $\Phi: X \to Y$ such that $\Phi(X) = Y$, $\Phi^{-1}(Y) = X$ and both the maps $\Phi, \Phi^{-1}$ are bornologous. A metric space $X$ is
• proper if each closed bounded subset of $X$ is compact;

• homogeneous if for any points $x, y \in X$ there is a bijective isometry $f: X \to X$ with $f(x) = y$.

**Theorem 1.** Each unbounded proper homogeneous metric space $X$ of asymptotic dimension zero is coarsely equivalent to the anti-Cantor set $\mathbb{Z}_2^\infty$.

The anti-Cantor set is the direct sum $\mathbb{Z}_2^\infty$ of countably many copies of the 2-element group $\mathbb{Z}_2$, endowed with the ultrametric

$$\text{dist}((x_i), (y_i)) = \max_i 2^i |x_i - y_i|.$$  

The anti-Cantor set is the asymptotic counterpart of the Cantor cube $\mathbb{Z}_2^\omega$ endowed with the ultrametric

$$\text{dist}((x_i), (y_i)) = \max_i 2^{-i} |x_i - y_i|.$$ 

It is known that each countable $G$ group carries a left-invariant metric turning $G$ into a proper metric space. Such a metric is unique up to a bijective coarse equivalence, see [3]. So, each countable group $G$ can be thought as a homogeneous proper metric space. This space is asymptotically zero-dimensional if and only if the group $G$ is locally finite in the sense that each finite subset of $G$ generates a finite subgroup, see [3]. Now we see that the above Theorem implies

**Theorem 2.** Each countable locally finite group is coarsely equivalent to the anti-Cantor set.

This theorem can be applied to derive the following coarse classification of countable Abelian groups.

**Theorem 1.** A countable Abelian group $G$ of asymptotic dimension $\text{asdim}(G) = n$ is coarsely equivalent

• to $\mathbb{Z}^n$ if $G$ is finitely generated and

• to $\mathbb{Z}^n \times \mathbb{Z}_2^\infty$ if $G$ is infinitely generated.

**References**

Pointwise Fuzzy Topology on Function Spaces in the Category of Intuitionistic Fuzzy Topological Spaces

Sadi Bayramov and Cigdem Gunduz

Department of Mathematics, Kafkas University, Kars, 36100, TURKEY
baysadi@gmail.com

In this study, we firstly give pointwise fuzzy topology on a given function space in the category of fuzzy bitopological spaces. By using this topology, we introduce and study pointwise intuitionistic fuzzy topology on a given function space in the category of intuitionistic fuzzy topological spaces.

**Theorem 1.** \( (Y^X, \sigma_p, \sigma^*_p) \) is intuitionistic fuzzy homeomorphic to the product

\[
\prod_{x \in X} (Y, \sigma_p, \sigma^*_p).
\]

**Theorem 2.** A map \( g : (Z, \tau, \tau^*) \rightarrow (Y^X, \sigma_p, \sigma^*_p) \) is a gp-map if and only if the map \( e_x \circ f : (Z, \tau, \tau^*) \rightarrow (Y, \sigma, \sigma^*) \) is a gp-map.

**Theorem 3.** If \( (Y, \sigma, \sigma^*) \) is strong Hausdorff, then \( (Y^X, \sigma_p, \sigma^*_p) \) is strong Hausdorff space for each \( (X, \tau, \tau^*) \).

**Theorem 4.** The mapping

\[
\nabla : \left( \prod_{x \in X} (Y_{s^t}^X, (\sigma^t_p, \sigma^*_p)) \right) \rightarrow (Y, \sum_{i=1}^n, \sigma^*, \sum^*_{i=1})
\]

is a fuzzy homeomorphism in the pointwise fuzzy topology.

**Theorem 5.** The mapping

\[
\Delta : \left( \prod_{x \in X} (Y_{s^t}^X, \prod_{t \in I} \sigma_{s^t p}, \prod_{t \in I} \sigma^*_{s^t p}) \right) \rightarrow \left( \prod_{t \in I} (Y_t^X, \prod_{t \in I} \sigma_t, \prod_{t \in I} \sigma^*_t) \right)
\]

is an intuitionistic fuzzy homeomorphism in the pointwise fuzzy topology.
Theorem 6. Let \((X, \tau, \tau^\ast), (Y, \sigma, \sigma^\ast)\) and \((Z, \eta, \eta^\ast)\) be IFTSs. For each function space \(Y^X\) with pointwise topology and for each gp-map \(g : X \to Y\),

\[ E^{-1}(g) : Z \times X \to Y \]

is a gp-map.

References


Convex Bodies of Constant Width in Spheres

Lidiya Bazylevych

National University “Lviv Polytechnica”, Lviv, UKRAINE
izar@littech.lviv.ua

The notion of convexity can be considered in arbitrary Riemannian manifold. We consider the standard Riemannian metric on the unit sphere \(S^2\) of \(\mathbb{R}^3\). Let \(U\) denote the open upper hemisphere of \(S^2\). We say that a subset \(A\) of \(U\) is convex if every two points in \(A\) can be joined with a geodesic in \(A\). We denote by \(cc(U)\) the hyperspace of compact convex subsets in \(U\). One can easily prove that the hyperspace \(cc(U)\) is homeomorphic to the contractible Q-manifold \(Q \setminus \{*\}\) (by \(Q\) we denote the Hilbert cube).

Note that a analogical result can be also obtained for the hyperspace of compact convex sets in the hyperbolic plane.

A convex set \(A\) in \(U\) is called a body of constant width \(d\), where we have \(d(a, b) \leq d\) and there exists a point \(c\) of the boundary of \(A\) such that \(d(a, c) = d\).

Denote by \(cw_d(U)\) the set of bodies of constant width \(d\) in \(U\). The main result states that this hyperspace is also homeomorphic to \(Q \setminus \{*\}\).

We leave as an open question whether an analogical result is valid for the hyperspace of compact convex bodies of constant width \(d\) in the hyperbolic plane \(\mathbb{H}^2\).
Topological Properties Preserved by Scatteredly Continuous Maps

Bohdan Bokalo and Taras Banakh

Ivan Franko National University of Lviv, UKRAINE
topology@franko.lviv.ua

In the talk we detect topological properties preserved by scatteredly continuous maps. A map $f: X \to Y$ between topological spaces is scatteredly continuous if for each non-empty subspace $A \subseteq X$ the restriction $f|A$ has a point of continuity. A bijective map $f: X \to Y$ is called a scattered homeomorphism if $f$ and $f^{-1}$ are scatteredly continuous.

We shall say that a class $\mathcal{P}$ of (regular) topological spaces is

- **topological** if a space $X$ belongs to $\mathcal{P}$ provided $X$ is homeomorphic to a space $Y \in \mathcal{P}$;

- **closed-hereditary** (resp. open-hereditary) if for each space $X \in \mathcal{P}$, every closed (resp. open) subspace $Y$ of $X$ belongs to $\mathcal{P}$;

- **(scatteredly) projective** if a (regular) space $Y$ belongs to $\mathcal{P}$ provided $Y$ is the image of a space $X \in \mathcal{P}$ under a (scatteredly) continuous map $f: X \to Y$;

- **$\sigma$-additive** if a (regular) space $X$ belongs to $\mathcal{P}$ if it has a countable closed cover $\mathcal{C} \subseteq \mathcal{P}$.

By the **additivity** $\text{add}(\mathcal{P})$ of a class $\mathcal{P}$ of (regular) spaces we understand the cardinal $\kappa$ for which there is a (regular) space $X \not\in \mathcal{P}$ that has a closed cover $\mathcal{C} \subseteq \mathcal{P}$ of size $|\mathcal{C}| \leq \kappa$. If no such a cardinal $\kappa$ exists, then we put $\text{add}(\mathcal{P}) = \infty$ and assume that $\kappa < \infty$ for all cardinals $\kappa$. Observe that $\mathcal{P}$ is $\sigma$-additive if and only if $\text{add}(\mathcal{P}) > \kappa$. By definition, for a topological $T_1$-space $X$ the large pseudocharacter $\Psi(X)$ is equal to the smallest cardinal $\kappa$ such that each closed subset $F \subseteq X$ can be written as the intersection $\cap \mathcal{U}$ of a family $\mathcal{U}$ consisting $\leq \kappa$ open subsets of $X$. It is easy to see that $\Psi(X) \leq \text{li}(X)$ for every regular space $X$. By the **paracompactness number** $\text{par}(X)$ of a topological space $X$ we understand the smallest cardinal $\kappa$ such that each open cover of $X$ can be refined by an closed cover $\mathcal{F}$ of $X$ that can be written as the union $\mathcal{F} = \bigcup_{\alpha \leq \kappa} \mathcal{F}_\alpha$ of $\kappa$ many locally finite families $\mathcal{F}_\alpha$ of closed subsets of $X$. Hence a topological space $X$ is paracompact if and only if $\text{par}(X) \leq 1$. 

Proposition 1. Let $\mathcal{P}$ be a closed-hereditary projective class of topological spaces and $f : X \to Y$ be a surjective scatteredly continuous map from a space $X \in \mathcal{P}$ onto a regular topological space $Y$. If $\max\{\par(X), \Psi(X)\} < \add(\mathcal{P})$, then $Y \in \mathcal{P}$.

Corollary 1. Let $f : X \to Y$ be a scatteredly continuous surjective map between regular spaces. If the space $X$ is hereditarily Lindelöf and $\sigma$-compact, then so is the space $Y$.

Corollary 2. A regular space $X$ is analytic if and only if it is the image of a Polish space $P$ under a scatteredly continuous map $f : P \to X$.

Now we detect some cardinal functions that respect scatteredly continuous maps. We define a cardinal function $\varphi$ on a closed-hereditary class $\mathcal{T}$ of topological spaces to be

- **topological-invariant** if $\varphi(X) = \varphi(Y)$ for any homeomorphic spaces $X, Y \in \mathcal{T}$;

- **additive** if $\varphi(X) \leq \sum_{C \in \mathcal{C}} \varphi(C)$ for any closed cover $\mathcal{C}$ of a space $X \in \mathcal{T}$;

- **closed-hereditary** (resp. **open hereditary**) if $\varphi(Y) \leq \varphi(X)$ for every closed (resp. open) subspace $Y$ of a space $X \in \mathcal{T}$;

- **(scatteredly) projective** if $\varphi(f(X)) \leq \varphi(X)$ for every scatteredly continuous map $f : X \to Y$ between spaces $X, Y \in \mathcal{T}$;

- **global** if $\varphi(D) \geq |D|$ for any discrete space $D \in \mathcal{T}$.

Observe that a cardinal function $\varphi$ on a closed-hereditary scatteredly projective class $\mathcal{T}$ is closed-hereditary, open-hereditary, projective, scatteredly projective if and only if for every cardinal $\kappa$ so is the class $\mathcal{P} = \{X \in \mathcal{T} : \varphi(X) \leq \kappa\}$.

Corollary 3. Let $\varphi$ be an additive, projective, and closed-hereditary cardinal function on the class of regular spaces. For any surjective scatteredly continuous map $f : X \to Y$ between regular spaces we get

$$\varphi(Y) \leq \max\{\varphi(X), \par(X), \Psi(X)\} \leq \max\{\varphi(X), hl(X)\}.$$  

Theorem 1. A global additive closed-hereditary cardinal function $\varphi$ on the class of regular spaces is scatteredly projective if and only if $\varphi$ is projective and $\varphi \geq hl$.

Corollary 4. If $f : X \to Y$ is a scatteredly continuous map between regular spaces, then
(i) $nw(Y) \leq nw(X)$;
(ii) $hl(Y) \leq hl(X)$;
(iii) $hd(Y) \leq \max\{hd(X), hl(X)\}$.

In fact, the second item of Corollary 4 holds for non-regular spaces too.

Because of an example of a regular space $X$ with $hd(X) < hl(X)$, Theorem 1 implies that the hereditary density $hd$ is not scatteredly-projective, which means that there is a scatteredly continuous map $f : X \rightarrow Y$ between regular spaces such that $hd(Y) > hd(X)$.

**Example.** Under the Diamond-Axiom A, Ostaszewski has constructed a regular space $X$ which is uncountable, compact, scattered, and hereditarily separable. Then any bijective map $f : X \rightarrow D$ to a discrete space $D$ is scatteredly continuous but $hd(D) = |D| = |X| > \aleph_0 = hd(X)$. This yields that the class of regular hereditarily separable spaces is not scatteredly projective under the Diamond-Axiom.

On the other hand, S. Todorcevic has constructed a model of ZFC without $S$-spaces, that is, regular hereditarily separable non-Lindelöf spaces. In such models the class of regular hereditarily separable spaces is scatteredly projective.

Next, we shall show that scatteredly homeomorphic spaces can be decomposed into a sum of closed homeomorphic subspaces.

**Theorem 2.** If $h : X \rightarrow Y$ is a scattered homeomorphism between regular spaces, then there are closed covers $\{X_i : i \in I\}$ and $\{Y_i : i \in I\}$ of $X$ and $Y$ for some index set $I$ of size

$$|I| \leq \max\{\par(X), \par(Y), \Psi(X), \Psi(Y)\}$$

such that for each $i \in I$ the restriction $h|X_i$ is a homeomorphism of $X_i$ onto $Y_i$.

Theorem 2 can be partly reversed. Namely, for scattered homeomorphisms a theorem of Cantor-Bernstein type holds.

**Theorem 3.** Two topological spaces $X, Y$ are scatteredly homeomorphic if each of then is homeomorphic to a closed subspace of the other space.

**Remark.** Remark that the Baire space $\mathbb{N}^\omega$ and the Cantor cube $2^\omega$ embed into each other, but fail to be scatteredly homeomorphic. The reason is that scattered homeomorphisms preserve the $\sigma$-compactness, see Corollary 2. This shows that the closedness is essential in Theorem 9.

**Theorem 4.** Let $\mathcal{P}$ be a closed-hereditary topological property.
1. A regular space $X$ with $\max\{\text{par}(X), \Psi(X)\} < \text{add}(\mathcal{P})$ has property $\mathcal{P}$ if and only if $X$ is scatteredly homeomorphic to a regular space $Y$ with property $\mathcal{P}$ and $\max\{\text{par}(Y), \Psi(Y)\} < \text{add}(\mathcal{P})$.

2. A regular space $X$ has property $\mathcal{P}$ if and only if $X$ is scatteredly homeomorphic to a regular space $Y$ with property $\mathcal{P}$ and $\text{hd}(Y) \leq \text{add}(\mathcal{P})$.

Applying this theorem to $\sigma$-additive topological properties, we get

**Corollary 5.** Let $\mathcal{P}$ be a closed-hereditary $\sigma$-additive topological property of perfectly paracompact spaces. A perfectly paracompact space $X$ has property $\mathcal{P}$ if and only if $X$ is scatteredly homeomorphic to a perfectly paracompact space $Y$ with property $\mathcal{P}$.

**Corollary 5.** If $X, Y$ are scatteredly homeomorphic perfectly paracompact spaces, then

(i) $\text{nw}(X) = \text{nw}(Y)$;
(ii) $\text{hd}(X) = \text{hd}(Y)$;
(iii) $\dim X = \dim Y$;
(iv) $X$ is $\sigma$-compact iff so is the space $Y$;
(v) $X$ is analytic iff so is the space $Y$.

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**Macroscopic Dimension of PSC-Manifolds**

Dmitry Bolotov

*B. Verkin Institute for Low Temperature Physics and Engineering, Lenina ave. 47, Kharkiv, UKRAINE, bolotov@univer.kharkov.ua*

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**The Topology of Saddle Manifolds**

Olesandr Borisenko

*Kharkiv National University, UKRAINE, borisenk@univer.kharkov.ua*
The Category Characterization of the Main Classes of the Uniform Spaces and Uniformly Continuous Mappings

Altay A. Borubaev

Erkindik, 2, Bishkek, KYRGYSTAN. n_expert@rambler.ru

In the talk the category characterization of the absolutes of the uniform spaces, compact, and complete uniform spaces, and uniformly perfect and complete mappings are obtained and investigated their some properties.

Luzin and Sierpinski Sets

Lev Bukovsky

Institute of Mathematics, P. J. Šafárik University, Košice, SLOVAKIA

Many modifications of the classical notions of a Luzin and a Sierpinski sets are investigated in literature. The famous result by Rothberger says that the existence of both classical Luzin and Sierpinski sets implies the continuum hypothesis. We present a general result of this kind for modified notions.

Embedding and Nonembedding Results in Asymptotic Geometry

Sergei Buyalo

St. Petersburg Department of Steklov Institute of Mathematics, RUSSIA
sbuyalo@pdmi.ras.ru

We present a survey on quasi-isometric embedding and nonembedding results in asymptotic geometry. A various constructions of quasi-isometric embeddings of (basically hyperbolic) metric spaces into other metric spaces will be discussed. In particular, we explain how any Gromov hyperbolic group $G$ can be embedded into $(n+1)$-fold product of binary metric trees,
where \( n \) is the topological dimension of the boundary at infinity of \( G \) (this result is due to S. Buyalo, V. Schroeder, A. Dranishnikov).

The results discussed in the first part of the talk are in a definite sense optimal. This is the topic of the second part of the talk, where we discuss some quasi-isometric invariants of metric spaces that serve as obstacles to quasi-isometric embeddings. Most important are the subexponential corank and the hyperbolic dimension. Open questions and some conjectures will be discussed.

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**Approximation of Dendroids by Trees**

Robert Cauty

*Département de Mathématiques, Université de Paris VI (Pierre et Marie Curie), FRANCE*

We prove that, for every dendroid \( X \), every tree \( T_0 \) contained in \( X \) and every \( \varepsilon > 0 \), there exists a tree \( T \), contained in \( X \) and containing \( T_0 \) and an \( \varepsilon \)-retraction of \( X \) onto \( T \). This implies that every dendroid is the limit of a projective sequence formed of trees and retractions.

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**Whitney Levels without Whitney Maps**

Wlodzimierz J. Charatonik and Jennifer Smith

*Missouri University of Science and Technology, USA,  wjcharat@umst.edu*

We propose a new definition of a Whitney level that does not require the existence of a Whitney map. This allows to investigate hyperspaces of non-metric continua in a similar way as we investigate metric ones. We will show examples of hyperspaces with many Whitney levels and others that do not admit any and we will show some Whitney properties for non-metric continua.
Inductive Dimensions Modulo $\mathcal{P}$

Vitalij A. Chatyrko and Y. Hattori

Linköping University, SWEDEN, vitja@mai.liu.se

All spaces are assumed to be separable and metrizable. Recently, we defined the spaces $S^Y_\alpha$, where $\alpha$ is any countable ordinal number and $Y$ is any space, by the use of construction of the Smirnov’s compacta $S^\alpha$. (In particular, if $Y$ is a one-point space then $S^Y_\alpha$ is the compactum $S^\alpha$). Then we applied the generalized Smirnov’s spaces $S^Y_\alpha$, where $\alpha$’s are countable ordinals and $Y$’s are special zero-dimensional subspaces of the closed interval $[0,1]$, for a complete description of the relationship between all transfinite dimensions modulo $\mathcal{P}$, $\mathcal{P}$-trInd, where $\mathcal{P}$ is any absolutely multiplicative or additive Borel class, on separable metrizable spaces. Now we extend some known results about the relationship between the transfinite dimensions trInd and trInd on Smirnov’s compacta $S^\alpha$ to the relationship between the transfinite dimensions modulo $\mathcal{P}$, $\mathcal{P}$-trInd and $\mathcal{P}$-trInd, where $\mathcal{P}$ is an absolutely multiplicative or additive Borel class, on the spaces $S^Y_\alpha$, where $Y$ is any finite-dimensional space.

On Order-Continuous set Multifunctions in Hausdorff Topology

Anca Croitoru and Alina Gavrilut

Faculty of Mathematics, "Al. I. Cuza" University of Iasi, Carol I Bd. No. 11, Iasi, 700506, ROMANIA, croitoru@uaic.ro

In this talk we present some properties of order-continuous (shortly, $o$-continuous) set multifunctions with respect to the Hausdorff topology and establish several decomposition theorems with respect to pseudo-atoms of $o$-continuous monotone multimeasures.
Complete Erdös Space and its Infinite Power

Jan Dijkstra and Jan van Mill

Vrije Universiteit, Amsterdam, NETHERLANDS

We discuss topological characterizations of the two spaces and their applications. We pay particular attention to factoring theorems, for instance, the infinite power of complete Erdös space is stable under multiplication with arbitrary complete almost zero-dimensional spaces.

On Some Properties of R. D. Kopperman
Stable Bitopological Spaces

Irakli Dochviri

Department of Applied Mathematics, Georgian Technical University, 0175 Tbilisi, 77 M. Kostava, GEORGIA, i.dochviri@gtu.ge

The asymmetric topology is one of the relatively new branches of modern mathematics. A central role in asymmetric topology play bitopologies, i.e. structures of the ordered pairs of topologies defined on same sets. Usually, an ordered triple \((X, \tau_1, \tau_2)\) is called to be a bitopological space [1]. Nowadays bitopological reasons are useful for computer science too. In this talk I present some of my results concerning to the stable bitopological spaces. Following [2], a bitopological space \((X, \tau_1, \tau_2)\) is called stable iff every nonempty closed set is compact relatively to the another topology. Using modified construction of A. Taimanov [3] we prove main result of this work. Recall that in the Theorem bellow is assumed that \(i, j \in \{1, 2\}\) and \(i \neq j\). Moreover, a single-valued map \(f: (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)\) is called \((i, j) - \Delta\) continuous if \(f: (X, \tau_1) \to (Y, \gamma_2)\) is continuous. For other notions used in the theorem see e.g. [4].

**Theorem.** Let a bispace \((X, \tau_1, \tau_2)\) be an \((i, j)\)-submaximal and \((Y, \gamma_1, \gamma_2)\) be \(p - E.D., (i, j) - QHC, (i, j)\)-stable, \(j\)-Urysohn bispace. Suppose that for a set \(A \in d - D(X)\) a map

\[ f: (A, \tau_1^*, \tau_2^*) \to (Y, \gamma_1, \gamma_2) \]

is \(i\)-open and \((i, j) - \Delta\) continuous. Then there exists an \((i, j) - \Delta\) continuous extension

\[ \varphi: (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2) \]
of the map $f$, iff for any pair of disjoint sets $F_1, F_2 \in \mathcal{D}$ we have

$$\tau_{\text{cl}} f^{-1}(F_1) \cap \tau_{\text{cl}} f^{-1}(F_2) = 0.$$ 

References


Quasilinear Operators and Morse Functions

Yakov Dymarskii

Lugansk State University of Internal Affairs, UKRAINE,
dymarsky@lep.1g.ua

Let $f$ be a smooth functional on a sphere. We consider the potential operator $F = \nabla f$ in quasilinear form: $F(y) = A(y)y$, where $A(y)$ is a self-adjointed operator. We shall show that critical points $y$ of the functional $f$ coincide with eigenvectors: $A(y)y = ky$ ($k$ is an eigenvalue). For K. Uhlenbeck manifolds we shall give a new geometric interpretation for Morse index.
Algebra in the Superextensions of Groups

Volodymyr Gavrylkiv and Taras Banakh

Precarpathian National University, UKRAINE, vgavrylkiv@yahoo.com

In the talk we shall discuss the properties of the semigroups of maximal linked systems. By definition, a family $\mathcal{L}$ of subsets of a set $X$ is called a linked system on $X$ if $A \cap B$ is nonempty for all $A, B \in \mathcal{L}$. Such a linked system is maximal linked if it coincides with any linked system $\mathcal{M}$ on $X$ that contains $\mathcal{L}$. For example, each ultrafilter is a maximal linked system.

The space $\lambda(X)$ of all maximal linked systems on $X$ is called the superextension of $X$, and is endowed with the topology generated by the sub-base consisting of the sets $U^+ = \{\mathcal{L} \in \lambda(X) : U \in \mathcal{L}\}$, where $U$ runs over subsets of $X$.

It is known that each binary operation $\ast$ on $X$ extends to a right topological operation on $\beta X$, the Stone–Čech compactification of $X$, playing a crucial role in Combinatorics of Numbers. In the same way the operation $\ast$ can be further extended to a right-topological operation on $\lambda(X)$ by the formula:

$$A \ast B = \{C \subseteq X : \{x \in X : x^{-1}C \in B\} \in A\}.$$

If the operation $\ast$ on $X$ is associative, then it extends to an associative operation on $\lambda(X)$. In this case $\beta X$ is a subsemigroup of $\lambda(X)$.

In the sequel $G$ is a group. We start with characterization the superextensions $\lambda(G)$ possessing (right) zeros.

**Theorem 1.** The superextension $\lambda(G)$ of a group $G$ possesses a right zero if and only if $G$ is odd in the sense that the order of each element of $G$ is odd.

**Theorem 2.** The superextension $\lambda(G)$ has a left zero if and only if $\lambda(G)$ has a zero if and only if $|G| \in \{1, 3, 5\}$.

**Theorem 3.** The superextension $\lambda(G)$ of a group $G$ is commutative if and only if $|G| \leq 4$.

Next, we describe cancellative elements of the superextensions. Recall that an element $x$ of a semigroup $S$ is right cancelable if for every $a, b \in X$ the equation $x \ast a = b$ has at most one solution $x \in S$.

We say that a maximal linked system $\mathcal{L} \in \lambda(G)$ (i) has finite support if there is a finite family $\mathcal{F} \subseteq \mathcal{L}$ of finite subsets of $G$ such that each set $L \in \mathcal{L}$ contains a set $F \in \mathcal{F}$; (ii) is free if for each $L \in \mathcal{L}$ and each finite subset $F \subseteq G$ the complement $L \setminus F$ belongs to $\mathcal{L}$.
Theorem 4. Let $G$ be a group. A maximal linked system $\mathcal{L} \in \lambda(G)$ is right cancelable in $\lambda(X)$ provided for every $x \in X$ there is a set $S_x \in \mathcal{L}$ such that the family $\{x + S_x : x \in X\}$ is disjoint.

Theorem 5. For each countable group $G$ the subsemigroup $\lambda^5(G)$ of free maximal linked systems contains an open dense subset consisting of right cancelable elements in the semigroup $\lambda(G)$.

By definition, the topological center of a right-topological semigroup $S$ is the set of all elements $a \in S$ such that the left shift $I_a : S \to S$, $I_a(x) = a \ast x$, is continuous.

Theorem 6. For any countable group $G$ the topological center of the semigroup $\lambda(G)$ coincides with the set $\lambda^*(G)$ consisting of all maximal linked systems with finite support.

Theorem 7. For any countable infinite group $G$ the algebraic center of $\lambda(G)$ coincides with the algebraic center of $X$.

For finite groups this theorem is not true.

Remark. The semigroup $\lambda(G)$ contains a central element distinct from a principal ultrafilter if $3 \leq |G| \leq 5$.

Finally, given an Abelian group $G$ we describe the structure of minimal left ideals of the superextension $\lambda(G)$. By $C_{2^k} = \{ z \in \mathbb{Z} : z^{2^k} = 1 \}$ we denote the cyclic group of order $2^k$. Let also $C_{2^{\infty}} = \bigcup_{k=1}^{\infty} C_{2^k}$ be the quasi-cyclic $2$-group. For a group $G$ by $q(G, C_{2^k})$ we denote the number of normal subgroups $H \subset G$ with quotient $G/H$ isomorphic to $C_{2^k}$. It is easy to see that for $k \in \mathbb{N}$

$$q(G, C_{2^k}) = \frac{b(G, C_{2^k}) - h(G, C_{2^{k-1}})}{2^{k-1}}$$

where $h(G, C_{2^k})$ is the number of homomorphisms from $G$ into $C_{2^k}$.

Theorem 8. For an Abelian group $G$ and an idempotent $e$ in the minimal ideal of $\lambda(G)$ the following conditions are equivalent:

1. $q(G, C_{2^\infty}) = 0$.

2. All the minimal left ideals are topological semigroups.

3. Some maximal subgroup of the minimal ideal of $\lambda(G)$ is compact.

4. All maximal subgroups of the minimal ideal of $\lambda(G)$ are topological groups.
5. The maximal subgroup $H(e) = e \cdot \lambda(G) \cdot e$ is topologically isomorphic to the Tychonoff product $\prod_{k=1}^{\infty} (C_{2^k})^{\lambda(G) \cdot C_{2^k}}$.

6. The set of idempotents of any minimal left ideal of $\lambda(G)$ is compact.

7. The set $E(\lambda(G) \cdot e)$ of idempotents of the minimal left ideal $\lambda(G) \cdot e$ is a compact semigroup of left zeros, homeomorphic to the cube $[0,1]^\lambda$ with

$$\lambda = \sum_{k=1}^{\infty} q(G, C_{2^k}) \cdot ((k+1)2^{k-1}-k-k).$$

8. The minimal left ideal $\lambda(G) \cdot e$ is topologically isomorphic to the product $H(e) \times E(\lambda(G) \cdot e)$.

9. The continuous homomorphism $\lambda(q) : \lambda(G) \rightarrow \lambda(G_2)$ induced by the pro-2-group reflexion $q : G \rightarrow G_2$ is injective on each minimal left ideal of $\lambda(G)$.

The last item of this theorem requires some explanations. The pro-2-group reflexion $q : G \rightarrow G_2$ is defined as follows. Consider the family $\mathcal{D}$ of normal subgroups $H$ of $G$ with $|G/H| = 2^k$ for some $k \in \mathbb{N}$. The quotient homomorphisms $q_H : G \rightarrow G/H$, $H \in \mathcal{D}$, compose a homomorphism $q : G \rightarrow \prod_{H \in \mathcal{D}} G/H$ to the Tychonoff product of finite 2-groups. The closure of $q(G)$ in $\prod_{H \in \mathcal{D}} G/H$ is denoted by $G_2$ and the homomorphism $q : G \rightarrow G_2$ is called the pro-2-group reflexion of $G$. For example, the pro-2-group reflexion $\mathbb{Z}_k$ of the group $\mathbb{Z}$ of integer is the group of integer 2-adic numbers. Since the compact group $G_2$ is the inverse limit of finite 2-groups $G/H$, $H \in \mathcal{D}$, its superextension $\lambda(G_2)$ in the inverse limit of the finite semigroups $\lambda(G/H)$, $H \in \mathcal{D}$, consequently, $\lambda(G_2)$ is a compact zero-dimensional topological semigroup. Now it is clear that the injectivity of the homomorphism $\lambda(q) : \lambda(G) \rightarrow \lambda(G_2)$ on a minimal left ideal of $\lambda(G)$ implies that this ideal is topologically isomorphic to a minimal left ideal in $\lambda(G_2)$ and thus is a topological semigroup.

References


Gottlieb Groups of Spheres and Projective Spaces

Marek Golasiński and Juno Mukai

Faculty of Mathematics and Computer Science, 87-100 Torun, Chopina 12/18, POLAND, marek@mat.uni.torun.pl

The Gottlieb groups $G_k(X)$ of a pointed space $X$ have been defined by Gottlieb in [2] and [1]: first $G_1(X)$ and then $G_k(X)$ for all $k \geq 1$. This is the subgroup of the $k$-th homotopy group $\pi_k(X)$ containing all elements which can be represented by a map $f: S^k \to X$ such that $\text{id}_X \vee f: X \vee S^k \to X$ extends (up to homotopy) to a map $F: X \times S^k \to X$, where $S^k$ is the $k$-sphere.

The higher Gottlieb groups $G_k(X)$ are related in [1] to the existence of sectioning fibrations with fiber $X$. For instance, if $G_k(X)$ is trivial then there is a cross-section for every fibration over the $(k+1)$-sphere $S^{k+1}$, with fiber $X$.

Basing on [1], we take up the systematic study of the Gottlieb groups $G_{n+k}(S^n)$ of spheres for $k \leq 13$ by means of the classical homotopy theory methods. We fully determine the groups $G_{n+k}(S^n)$ for $k \leq 13$ except for the 2-primary components in the cases: $k = 9$, $n = 53$; $k = 11$, $n = 115$.

Next, by use the classical results of homotopy groups of spheres and Lie groups, we determine some Gottlieb groups of projective spaces or give the lower bounds of their orders.

References


Inverse System and Inverse Limit of Intuitionistic Fuzzy Topological Spaces

Cigdem Gunduz (Aras)

Department of Mathematics, Kocaeli University, 41380, Kocaeli, TURKEY
carasgunduz@gmail.com, caras@kou.edu.tr

We study inverse system and inverse limit in the category of intuitionistic fuzzy topological spaces and series of their properties. Let $IFTS$ be the category of intuitionistic fuzzy topological spaces and $J$ be a direct poset (consider as a category).

Definition 1. Any functor $D: J^{op} \rightarrow IFTS$ is called an inverse system in $IFTS$, the limit of $D$ is called an inverse limit of $D$.

Theorem 2. Every inverse system in the category of $IFTS$ has a limit, and this limit is unique.

Theorem 3. Let $Inv(IFTS)$ be a category of all inverse systems in $IFTS$ and all mappings between them. Then $\lim \bigwedge$ operation is a functor from the category of $Inv(IFTS)$ to the category of $IFTS$.

Lemma 4. Let $f: (I^X, \tau) \rightarrow (I^Y, \sigma)$ be a mapping of $IFTS$s.

$f$ is an intuitionistic fuzzy open (closed) gp-map if and only if $f: (I^X, \tau') \rightarrow (I^Y, \sigma')$ is fuzzy open (closed) for each $r \in I_0$.

Theorem 5. Let $f = \left( \varphi: J' \rightarrow J, \left\{ f_{i'}: I^{X_i(\varphi)} \rightarrow I^{Y_{i'}} \right\}_{i' \in J'} \right)$ be a morphism from the inverse system $X = \left\{ I^{X_i} \right\}_{i \in J}$ to the inverse system $Y = \left\{ I^{Y_{i'}} \right\}_{i' \in J'}$ in the category of $Inv(IFTS)$. If $f_{i'}$ is an injective (bijective) gp-map for each $i' \in J'$, then $\lim f = \lim X \rightarrow \lim Y$ is an injective (bijective) gp-map.

Theorem 6. If $\left\{ \left\{ I^{X_i}, \tau_i \right\}_{i \in J}, \left\{ I^{Y_i}, \sigma_i \right\}_{i \in J'} \right\}$ is an inverse system of fuzzy compact Hausdorff spaces, then $\lim I^{X_i}$ is a fuzzy compact space.

References
Topological Cancellative Semigroups

Igor Guran

Ivan Franko National University of Lviv, UKRAINE
topology@franko.lviv.ua

Let $S$ be a topological cancellative semigroup and $S = T \bigcup G$ be its decomposition into the maximal ideal and maximal group. We consider properties of the topological semigroup $S$ in dependence on the properties of the maximal ideal under the assumption of compactness of the maximal group $G$.

On Linearly Ordered $H$-Closed Topological Semilattices

Oleg Gutik and Dušan Repovš

Ivan Franko National University of Lviv, UKRAINE
e.gutik@franko.lviv.ua

In our report all topological spaces will be assumed to be Hausdorff. We shall follow the terminology of [1, 2].

A topological semigroup $S$ is called $H$-closed, if $S$ is a closed subsemigroup of any topological semigroup $T$ which contains $S$ both as a subsemigroup and as a topological space [3].

A linearly ordered topological semilattice $E$ is called complete if every non-empty subset of $S$ has inf and sup.

We give a criterium when a linearly ordered topological semilattice is $H$-closed.

**Theorem 1.** A linearly ordered topological semilattice $E$ is $H$-closed if and only if the following conditions hold:

(i) $E$ is complete;

(ii) $x = \sup A$ for $A = \downarrow A \setminus \{x\}$ implies $x \in \text{cl}_E A$, whenever $A \neq \emptyset$; and

(iii) $x = \inf B$ for $B = \uparrow B \setminus \{x\}$ implies $x \in \text{cl}_E B$, whenever $B \neq \emptyset$. 
A topological semigroup $S$ is called *absolutely H-closed*, if any continuous homomorphic image of $S$ into a topological semigroup $T$ is $H$-closed [4].

**Corollary.** Every linearly ordered $H$-closed topological semilattice is absolutely $H$-closed.

**Theorem 2.** Every linearly ordered topological semilattice is a dense sub-semilattice of an $H$-closed linearly ordered topological semilattice.

**References**


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**The Cell-Like Approximation Theorem in Dimension 5**

**Denise Halverson** and **Robert J. Daverman**

*Brigham Young University, U.S.A., deniseh@math.byu.edu*

An overview of the proof of Edward’s Cell-like Approximation Theorem in dimension 5 will be presented. The Cell-like Approximation Theorem characterizes the $n$-manifolds precisely as the resolvable ENR homology $n$-manifolds with the Disjoint Disks Property for $5 \leq n < \infty$. The case $n = 5$ requires more care than the higher dimensional cases, the details of which have been published only recently.
Approximation of Capacities

Inna Hlushak and Oleh Nykyforchyn

Precarpathian National University, Ivano-Frankivsk, UKRAINE
inna.g1@rambler.ru

In this paper we investigate optimal approximations by $\cup$-capacities (or $\cap$-capacities) and optimal approximations by capacities on a closed subspace $X_0 \subseteq X$.

On the space of upper semicontinuous capacities [3] on a metric compactum $(X,d)$ we consider the following metric:

$$d(c,c') = \inf \{ \varepsilon > 0 \mid \forall F \subseteq X \ c(O_{\varepsilon}(F)) + \varepsilon \geq c'(F), c'(O_{\varepsilon}(F)) + \varepsilon \geq c(F) \}.$$  

For a point $x$ and a subset $A$ of a metric space $X$ we denote by $B_{\varepsilon}(x)$ and $O_{\varepsilon}(x)$ the closed $\varepsilon$-ball with the center $x$ and the closed $\varepsilon$-neighborhood of the set $A$.

We call a capacity $c \in MX$ a $\cap$-capacity (necessity measure) [2] if for all $A, B \subseteq X$ we have $c(A \cap B) = \min \{ c(A), c(B) \}$. Analogously, we call a capacity $c \in MX$ a $\cup$-capacity (also possibility measure [2]), if for all $A, B \subseteq X$ we have $c(A \cup B) = \max \{ c(A), c(B) \}$.

We denote the set of all $\cap$-capacities ($\cup$-capacities) on a compactum $X$ as $M_\cap X$ ($M_\cup X$). It is proved [2, 3] that the space $MX$ and its subspaces $M_\cap X$ and $M_\cup X$ are compacta.

If $c$ is a capacity on $X$, then the function $\kappa_X(c)$ that is determined on the set of all closed subsets $X$ by the formula $\kappa_X(c)(F) = 1 - c(X \setminus F)$ is a capacity as well. It is called dual or conjugate to $c$. It is also proved in [2] that a capacity $c$ on a compactum $X$ is a $\cap$-capacity if and only if the dual capacity $\kappa_X(c)$ is a $\cup$-capacity.

Each capacity $c_0$ on a closed subspace $X_0$ of a space $X$ can be extended to $X$ by the formula $c(F) = c_0(F \cap X_0)$, $F \subseteq X$.

We show that the distance from a capacity $c$ to the subspace $M_\cup X$ is equal to the infimum of all $\varepsilon \geq 0$ such that the following condition holds:

$$c(X \setminus \overline{O_{\varepsilon}(B_{\varepsilon}^\alpha)}) \leq \alpha + \varepsilon \text{ for } \alpha \in I,$$  

where $B_{\varepsilon}^\alpha = \{ x \in X \mid c(B_{\varepsilon}(x)) \geq \alpha - \varepsilon \}$.

Here is an algorithm that finds one of $\cup$-capacities that are nearest to $c$: we find the least $\varepsilon \geq 0$ such that $(\ast)$ is valid and use the obtained sets.
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$B^n_0$ to construct a capacity $c_0 \in M_0 X$ that is an optimal approximation (in fact it is the greatest of optimal approximations). The capacity $c_0$ is of the form:

$$c_0(F) = \sup \{ \alpha \in I \mid F \cap B^n_0 \neq \emptyset \}$$

for any closed subset $F$ of $X$.

The map $\kappa X$ is an isometry of $(M X, \hat{d})$ onto itself, therefore the capacity $c'_0 \in M_0 X$ that is nearest to a capacity $c \in M X$ can be found as follows: we find the capacity $c_0 \in M_0 X$ which is nearest to $\kappa X(c)$ and then we go to the dual capacity: $c'_0 = \kappa X(c_0)$.

It is also proved that the distance from a capacity $c$ on $X$ to a subspace $M X_0 \subseteq M X$ is equal to

$$\varepsilon = \min \{ \varepsilon \geq 0 \mid c(\overline{O}_2(X_0)) \geq 1 - \varepsilon, c(X \setminus \overline{O}_2(X_0)) \leq \varepsilon \}.$$ 

One of nearest to $c \in M X$ capacities on $X_0$ is defined by the formula:

$$c_\varepsilon = \begin{cases} 
\min(c(\overline{O}_2(F)) + \varepsilon, 1), & F \neq \emptyset, \\
0, & F = \emptyset.
\end{cases}$$

A capacity $c_0 \in M X_0$ is one of nearest to $c$ if and only if

$$\kappa X(\kappa X(c)_{\varepsilon}) \leq c_0 \leq c_{\varepsilon}.$$ 

references


Quotient Topologies on (Boolean) Topological Semigroups

Olena Hryniv

Ivan Franko National University of Lviv, UKRAINE

It is well known that for a closed normal subgroup $H$ of a topological group $G$ the quotient group $G/H$ endowed with the quotient topology (that is the strongest topology making the quotient homomorphism $q: G \to G/H$ continuous) is a topological group too. In the category of topological semigroups such a result is not valid even in the simplest case of the quotient semigroup $S/I$ of a topological semigroup $S$ by a closed ideal $I \subseteq S$, see [1, 2]. Nonetheless if $I$ is a closed ideal in a locally compact $\sigma$-compact topological semigroup $S$, then the quotient semigroup $S/I$ is a topological semigroup [1] (see also [2]). In fact, the $\sigma$-compactness of $S$ in this result can be replaced by the $\sigma$-compactness of $I$:

**Theorem 1.** For any closed $\sigma$-compact ideal $I$ in a locally compact topological (inverse) semigroup $S$ the quotient $G/I$ is a topological (inverse) semigroup.

The $\sigma$-compactness of $I$ is essential in this theorem because of the following

**Example.** There is a Boolean locally compact topological semigroup $S$ and a closed discrete ideal $I \subseteq S$ such that the intersection $I \cap E$ with the set $E$ of idempotents of $S$ is compact but the quotient $S/I$ is not a topological semigroup.

A semigroup $S$ is *Boolean* if it is commutative and $xxx = x$ for all $x \in S$. Each Boolean semigroup $S$ is an inverse Clifford semigroup, that is the union $S = \bigcup_{e \in E} H(e)$ of subgroups $H(e) = \{x \in S : xx = e\}$ parameterized by idempotents $e \in E = \{x \in S : xx = x\}$.

The semigroup $S$ from the above example is the product $E \times H$ of the convergent sequence $E = \{0\} \cup \{1/n : n \in \N\}$ endowed with the operation of minimum and the free Boolean group $H$ over a discrete uncountable space, and $I = \{0\} \times H$.

**References**

Some Questions from the Uniformly Continuous Mappings Theory

Bekbolot E. Kanetov and Tumar J. Kasymova

Mathematics, Informatics and Cybernetics Faculty, Kyrgyz National University named after J. Balassagyn, Bishkek, KYRGYSTAN, tumar2000@mail.ru

In this work the uniformly paracompact mappings of uniform spaces have been studied. It has been proved that the uniform R-paracompact properties are preserved in preimage.

Michael’s Problem and Weakly Infinite-Dimensional Spaces

Alexandre Karassev

Nipissing University, CANADA, alexandk@nipissingu.ca

The aim of this paper is to introduce and study the concept of CSO-compact space via the notation of simply-open sets as well as to investigate their relationship to some well known classes of topological spaces and give some of its properties.
The Functor of Non-Expanding Functionals is not Open

Lesya Karchevs’ka and Taras Radul

*Ivan Franko National University of Lviv, UKRAINE, tarasradul@yahoo.co.uk*

The functor $E$ of non-expanding functionals was introduced by A. Stan’ko and J. Camargo. It contains many known functors: hyperspace exp, space of probability measures $P$, superextension $\lambda$, space of hyperspaces of inclusion $G$, the functor of order-preserving functionals $O$ and many other as subfunctors. All the functors mentioned above are open and hence bicommutative (see for example [1]).

**Theorem.** *The functor $E$ is finitely open but fails to be open (moreover, $E$ is not bicommutative).*

A functor $F$ is called *(finitely) open* if for any surjective open map $f: X \to Y$ between (finite) compact spaces the map $Ff : FX \to FY$ is open.

**References**


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On Extension of Continuous Mappings to $F_\sigma$-Measurable Mappings

Olena Karlova

*Chernivtsi National University, UKRAINE, mathan@ukr.net*

A mapping $f : X \to Y$ from a topological space $X$ to a topological space $Y$ is called *$F_\sigma$-measurable* if for every open set $V$ in $Y$ the preimage $g^{-1}(V)$ is an $F_\sigma$-set in $X$.

K. Kuratowski [1] proved that if $X$ is a metric space, $Y$ is a Polish space then every continuous mapping $f : E \to Y$, $E \subseteq X$, can be extended to an $F_\sigma$-measurable mapping $g : X \to Y$.

The following result was established in [2]: let $E$ be a Lindelöf subspace of a completely regular space $X$, $Y$ be a Polish space and either $E$ be
hereditarily Baire, or $E$ be $G_δ$ in $X$, then every $F_σ$-measurable mapping $f : E \to Y$ can be extended to an $F_σ$-measurable mapping $g : X \to Y$.

We prove that any continuous mapping $f : E \to Y$ on a completely metrizable subspace $E$ of a perfect paracompact space $X$ can be extended to an $F_σ$-measurable mapping $g : X \to Y$ with values in an arbitrary topological space $Y$.

References


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Some Questions from the Uniformly Continuous Mappings Theory

Tumar J. Kasymova and Bekbolot E. Kanetov

Mathematics, Informatics and Cybernetics Faculty, Kyrgyz National University named after J. Balkasagyn, Bishkek, KYRGYSZTAN, tumar2000@mail.ru

In this work uniform paracompact mappings of uniform spaces have been studied. It has been proved that the uniform paracompact properties are preserved in preimage. And some properties the compactness and completeness type have been studied.

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On the Space of Quotient Objects of Compact Hausdorff Spaces

K. Koporh

Pecarpathian National University, Ivano-Frankivs’k, UKRAINE

Let $X$ be a compact Hausdorff space. Following E. Shchepin [1] we say that two continuous onto maps $f_i : X \to Y_i$, $i = 1, 2$, are equivalent (written $f_1 \sim f_2$) provided that there exists a homeomorphism $h : Y_1 \to Y_2$ such that $f_2 = hf_1$ (here $Y_1, Y_2$ are also compact Hausdorff spaces).
The set $\Phi(X) = \{ [f] \mid f \text{ is an onto map of compact Hausdorff spaces} \} \text{ of equivalence classes can be topologized in different ways. This can be done by identifying every } [f] \in \Phi(X), \text{ for } f : X \to Y, \text{ with the set of } f_*(C(Y)) \in \text{CL}(C(X)), \text{ where CL}(C(X)) \text{ stands for the set of nonempty closed subsets of the Banach space } C(X) \text{ of the continuous functions on } X, \text{ and to topologize } \Phi(X) \text{ with one of different topologies on the space CL}(C(X)). \text{ In the sequel, we consider the Wijsman topology on the set CL}(C(X)) \text{ (see [2]).}$

It turns out, however, that this construction is not functorial on the category Comp of compact Hausdorff spaces.

However, if $i : X \to Y$ is an a embedding of $X$ as an open subset of $Y$, then the induced map $\Phi(X) \to \Phi(Y)$ is continuous. The proof of this is based on the fact that the natural map $m: \Phi(X_1) \times \Phi(X_2) \to \Phi(X_1 \sqcup X_2)$ is continuous.

By $\Phi_0(X)$ we denote the set of all equivalence classes of open subsets of $X$. Every element $[f]$ of the set $\Phi_0(X)$ is identified with the family $\{ f^{-1}(f(x)) \mid x \in X \} \in \exp^2(X)$ and is topologized by the Vietoris topology on $\exp^2(X)$.

We prove that this construction is functorial on the category Comp of compact Hausdorff spaces and open surjective maps.

References


FCA-Like Approach to Computational Quantum Topology

Martin M. Kovár

University of Technology in Brno, CZECH REPUBLIC, kovar@feec.vutbr.cz

We introduce and investigate topology-like structures (called frameworks) which may be represented as special formal contexts in formal concept analysis (FCA, founded by B. Ganter and R. Wille). We also study the
possibility of approximation of general topological spaces by finite frameworks. Obtained results could be useful for study certain relationships in quantum topology and topological causality structures, motivated by research in quantum gravity.

Generalized Cantor Manifolds in Finite and Infinite Dimension Theories

Pawel Krupski, A. Karassev, V. Todorov and V. Valov

University of Wroclaw, POLAND, krupski@math.uni.wroc.pl

A classical theorem of Alexandroff states that every n-dimensional compactum X contains an n-dimensional Cantor manifold. This theorem has a number of generalizations obtained by various authors. We consider extension-dimensional and infinite dimensional analogs of strong Cantor manifolds, Mazurkiewicz manifolds, and V*-continua, and prove corresponding versions of the above theorem.

Continuous Maps that Reduce Inductive Dimensions

Jerzy Krzempek

Silesian University of Technology, POLAND, j.krzempek@polsl.pl

It is well-known that the theorem on dimension-lowering maps for Ind is not true even for continuous maps between compact spaces. We present a new series of counter-examples in this compact case.

We modify a recent construction by V.A. Chatyrko, and for every pair of natural numbers $m > n \geq 1$, obtain a compact space $X_{m,n}$ such that

(a) $\text{ind } X_{m,n} = \text{Ind } X_{m,n} = m$, and

(b) every component of $X_{m,n}$ is homeomorphic to the $n$-dimensional cube $I^n$. 
It follows that \( \dim X_{m,n} = n \). If we consider the decomposition \( D \) of \( X_{m,n} \) into components, then the quotient space \( X_{m,n}/D \) is zero-dimensional, and for the quotient map \( f: X_{m,n} \to X_{m,n}/D \), we have

\[
m = \text{Ind}X_{m,n} > \text{Ind}X_{m,n}/D + \text{Ind}f = n.
\]

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**Inverse Systems and I-Favorable Spaces**

*Andrzej Kucharski*

*Institute of Mathematics, University of Silesia, Katowice, POLAND*  
akuchar@ux2.math.us.edu.pl

We show that a compact space is I-favorable if, and only if it can be represented as the limit of a \( \sigma \)-complete inverse system of compact metrizable spaces with skeletal bonding maps. We also show that any completely regular I-favorable space can be embedded as a dense subset of the limit of a \( \sigma \)-complete inverse system of separable metrizable spaces with skeletal bonding maps.

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**Sparse, Thin and \( P \)-Small Subsets of Infinite Groups**

*E. Lutsenko* and I. V. Protasov

*Department of Cybernetics, Kyiv National University, UKRAINE*  
protasov@unicyb.kiev.ua

A subset \( A \) of an infinite group \( G \) with the identity \( e \) is said to be

- **thin** if \( gA \cap A \) is finite for every \( g \in G, g \neq e \);
- **k-thin** for \( k \in \mathbb{N} \) if \( |gA \cap A| \leq k \) for each \( g \in G, g \neq e \);
- **almost thin** if \( \Delta(A) = \{ g \in G : gA \cap A \text{ is infinite} \} \) is finite;
- **sparse** if, for every infinite subset \( X \subseteq G \), there exists a non-empty finite subset \( F \subseteq X \) such that \( \bigcap_{g \in F} gA \) is finite;
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- **k-sparse** for $k \in \mathbb{N}$ if, for every infinite subset $X$ of $G$, there exists a non-empty finite subset $F$ of $X$ such that $|F| \leq k$ and $\bigcap_{g \in F} gA$ is finite;

- **$P$-small** if there exists an injective sequence $(g_n)_{n \in \omega}$ in $G$ such that the subsets $(g_nA)_{n \in \omega}$ are pairwise disjoint;

- **almost $P$-small** if there exists an injective sequence $(g_n)_{n \in \omega}$ in $G$ such that $g_i A \cap g_j A$ is finite for all distinct $i, j \in \omega$.

**Theorem 1.** For every group $G$ and every $k \in \mathbb{N}$, there exists a $(k + 1)$-sparse but not $k$-sparse subset of $G$.

**Theorem 2.** For every group $G$, there exists a sparse subset of $G$ which is not $k$-sparse for every $k \in \omega$.

**Theorem 3.** For every group $G$, there exists a thin subset of $G$ which is not $k$-thin for every $k \in \omega$.

**Theorem 4.** Every group $G$ can be generated by some 2-thin subset.

**Theorem 5.** Every almost thin subset $A$ of a group $G$ can be partitioned in $2^{|A|^{1/3}}$ thin subsets. If $G$ has no elements of odd order, then $A$ can be partitioned in $2^{|A|^{1/3}}$ thin subsets.

**Theorem 6.** Every almost thin subset of a group $G$ is 2-sparse. Every 2-sparse subset of a group $G$ is almost $P$-small. Every almost $P$-small subset can be partitioned in two $P$-small subsets.

**Theorem 7.** For every group $G$, there exists a 2-sparse subset which cannot be partitioned in finitely many thin subsets.

**Theorem 8.** For every group $G$, there exists a 2-thin subset which is not $P$-small.

**Theorem 9.** For every group $G$, there exists a $P$-small subset which is not sparse.

**Theorem 10.** If a subset $A$ of a group $G$ is either sparse or almost $P$-small, then $\mu(A) = 0$ for each left invariant Banach measure $\mu$ on $G$. 
Constructing Small Subsets with a Given Packing Index in Abelian Groups

Nadia Lyaskovska

Ivan Franko National University of Lviv, UKRAINE

For a subset $A$ of a group $G$, we consider the following two cardinal numbers

$$ind_p(A) = \sup\{|S| : S \subseteq G \text{ is such that } \{xA\}_{x \in S} \text{ is disjoint}\}$$

and

$$Ind_p(A) = \sup\{|S| : S \subseteq G \text{ is such that } \{xA\}_{x \in S} \text{ is almost disjoint}\},$$

called the packing indices of $A$ in $G$.

Those packing indices have sharp versions carrying a bit more information about $A$:

$$ind_p^{\dagger}(A) = \sup\{|S|^{\dagger} : S \subseteq G \text{ is such that } \{xA\}_{x \in S} \text{ is disjoint}\}$$

and

$$Ind_p^{\dagger}(A) = \sup\{|S|^{\dagger} : S \subseteq G \text{ is such that } \{xA\}_{x \in S} \text{ is almost disjoint}\}.$$

It follows that $ind_p(A) = \sup \{\kappa : \kappa < ind_p^{\dagger}(A)\}$ and $ind_p(A) \leq Ind_p(A)$.

The (sharp) packing indices measure the geometric smallness of a subset of a group. Subsets with small packing index can be thought as large in a geometric sense.

**Theorem 1.** Let $G$ be an infinite Abelian group and $L \subseteq G$ be a subset with $Ind_p(L) = 1$. For a cardinal $\kappa \in [2, |G|^{\dagger}]$ the following conditions are equivalent:

1) there is a subset $A \subseteq G$ with $ind_p^{\dagger}(A) = \kappa$;

2) there is a subset $A \subseteq L$ with $ind_p^{\dagger}(A) = Ind_p^{\dagger}(A) = \kappa$;

3) if $|G/|G|_2| \leq 2$, then $\kappa \neq 4$ and if $G = [G]_3$, then $\kappa \neq 3$.

Here $[G]_p = \{x \in G : x^p = 1\}$ stands for the subgroup of elements of order $p$. 
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Observe that the equality $\text{Ind}_P(L) = 1$ is equivalent to $|L \cap (g+L)| = |G|$ for all $g \in G$, which means that $L$ is rather large in a geometric sense. Still such a set $L$ can be presented as union of small sets.

**Theorem 2.** Each infinite group $G$ contains two subsets $A, B \subset G$ such that $\text{ind}_P^+(A) = \text{ind}_P^+(B) = |G|^+$ but $\text{Ind}_P(A \cup B) = 1$.

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**The Space of Real Places of a Field**

Michal Machura

*Institute of Mathematics, University of Silesia, Katowice, POLAND*

machura@ux2.math.us.edu.pl

A real place of a field $K$ is map $f : K \to \mathbb{R} \cup \{\infty\}$ preserving the algebraic operations in the sense that the equalities $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$ hold whenever $f(x) + f(y)$ or $f(x)f(y)$ is defined. In the talk we shall discuss the topological structure of the (compact Hausdorff) space $M_K$ of real places of a field $K$. We will examine the case when $K$ is an algebraic extension of the field of rational functions $R(x)$, and will give several examples of spaces of real places, homeomorphic to the circle or the closed interval. It will be also shown that the space of real places of the field of rational functions $R(x, y)$ projects onto the Pontryagin surface and thus is not metrizable.

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**Functions with Isolated Singularities on Surfaces**

Sergiy Maksymenko

*Institute of Mathematics, NAS of Ukraine, Terehoshenkovs’ka str., 3, Kyiv, 01601, UKRAINE, maks@imath.kiev.ua*

Let $M$ be a smooth connected compact surface, $P$ be either a real line $\mathbb{R}^1$ or a circle $S^1$, and $f : M \to P$ a smooth mapping. We introduce five axioms for $f$ under which one is able to describe the homotopy types of connected components of the stabilizers and orbits of $f$ with respect to the right and left-right actions of the groups of diffeomorphisms of $M$ and $P$. This result extends the analogous calculations concerning Morse functions [1,2,5] and is based on the results of [3,4].

In particular, suppose that $f : M \to P$ has the following properties:
(1) $f$ is constant on every connected component of $\partial M$ and $\Sigma_f \subset Int(M)$;

(2) for every critical point $z \in \Sigma_f$ there is a local presentation $f : \mathbb{R}^2 \to \mathbb{R}^1$ of $f$ such that $z = (0,0) \in \mathbb{R}^2$ and $f(x,y)$ is a homogeneous polynomial of some degree $p_x \geq 2$ without multiple factors.

Then $f$ satisfies the mentioned above axioms.
Also notice that $p_x = 2$ for all $z \in \Sigma_f$, then $f$ is Morse and the calculations of the homotopy types are given in [1,2,5].

References


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Discontinuities of A-Continuous Functions

Oleksandr Maslyuchenko

Chernivtsi National University, UKRAINE, mathan@ukr.net

A map $\mathcal{A} : X \ni x \mapsto A_x \in 2^{2^X} \setminus \{\emptyset\}$ is called a structure on a set $X$. As expected, a structure $\mathcal{A}$ on $X$ is called topological if for every $x \in X$ the following four axioms are satisfied:

$(N_1)$ $x \in A$ for every $A \in \mathcal{A}_x$;

$(N_2)$ if $A \in \mathcal{A}_x$ and $B \supseteq A$, then $B \in \mathcal{A}_x$;

$(N_3)$ if $A, B \in \mathcal{A}_x$ then $A \cap B \in \mathcal{A}_x$;
(N_4) for any \( A \in \mathcal{A}_x \), there is \( B \in \mathcal{A}_y \) such that \( A \in \mathcal{A}_y \) for all \( y \in B \).

A structure \( \mathcal{A} \) is called quasitopological (pretopological) if the conditions \( N_1, N_2 \) (and \( N_3 \)) hold.

Let \( \mathcal{A} \) be a structure on a set \( X \) and \( Y \) be a topological space. A map \( f : X \rightarrow Y \) is defined to be \( \mathcal{A} \)-continuous at a point \( x \in X \) if for every neighborhood \( V \) of the point \( f(x) \) there is \( A \in \mathcal{A}_x \) such that \( f(A) \subseteq V \) (a similar notion has been considered in [1]). We say that \( f \) is \( \mathcal{A} \)-continuous if it is \( \mathcal{A} \)-continuous at each point \( x \in X \). Each structure \( \mathcal{A} \) generates the quasitopological structure \( \mathcal{B}_x = \{ B \subseteq X : x \in B \text{ and } A \subseteq B \text{ for some } A \in \mathcal{A}_x \} \) so that the \( \mathcal{A} \)-continuity is equivalent to the \( \mathcal{B} \)-continuity. The elements of the system \( \mathcal{B}_x \) are called \( \mathcal{A} \)-neighborhoods of \( x \). A set \( E \subseteq X \) is called \( \mathcal{A} \)-joint if there are sets \( A \) and \( B \) such that \( cl(A) \cap B = A \cap cl(B) = \emptyset \), \( cl(A) \cap cl(B) = E \) and \( cl(A) \) is an \( \mathcal{A} \)-neighborhood of each point \( x \in E \). We say that \( E \) is \( \sigma \)-\( \mathcal{A} \)-joint if there is a sequence of \( \mathcal{A} \)-joint sets \( E_n \) such that \( E = \bigcup_{n=1}^{\infty} E_n \).

**Theorem 1.** Let \( X \) be a metrizable space, \( Y \) be a separable metrizable space, \( \mathcal{A} \) be a structure on \( X \) and \( f : X \rightarrow Y \) be a quasicontinuous \( \mathcal{A} \)-continuous map of the first Baire class. The set \( D(f) \) of discontinuity points of \( f \) is \( \sigma \)-\( \mathcal{A} \)-joint.

**Theorem 2.** Let \( X \) be a metrizable space, \( \mathcal{A} \) be a pretopological structure on \( X \) and \( E \subseteq X \) be a \( \sigma \)-\( \mathcal{A} \)-joint set. Then there is a lower semicontinuous quasicontinuous \( \mathcal{A} \)-continuous function \( f : X \rightarrow \mathbb{R} \) such that \( D(f) = E \).

**Theorem 3.** Let \( X \) be a metrizable space, \( \mathcal{A} \) be a pretopological structure on \( X \). For a subset \( E \subseteq X \) the following conditions are equivalent:

(i) \( E \) is \( \sigma \)-\( \mathcal{A} \)-joint;

(ii) there is a semicontinuous quasicontinuous \( \mathcal{A} \)-continuous function \( f : X \rightarrow \mathbb{R} \) with \( D(f) = E \);

(iii) there is a quasicontinuous \( \mathcal{A} \)-continuous first Baire class function \( f : X \rightarrow \mathbb{R} \) with \( D(f) = E \).

**References**

Uncountable Absorbing Systems in a Functional Space Related to the Hausdorff Dimension

Natalia Mazurenko

Precarpathian National University, Ivano-Frankiv’sk, UKRAINE
anatali@ukr.net

We are interested in description of topology of some uncountable system of sets of continuous functions on the \( n \)-dimensional cube \( \mathbb{I}^n \), where for these functions the Hausdorff dimension of their graphs takes the fixed values from the certain ordered set (from obvious reasons it follows that such dimension can take the values from an interval \([n, n + 1]\)).

If \( A \) is a countable infinite set then we denote \( s(A) = (-1, 1)^A \) (observe that \( s(A) \cong s \), where \( s = (-1, 1)^\mathbb{N} \) is the pseudo-interior of the Hilbert cube \( Q = [-1, 1]^\mathbb{N} \) and define the following subspace of \( s(A) \): \( \Sigma(A) \) consists of all \( x = (x_a)_{a \in A} \) in \( s(A) \) such that \( \sup_{a \in A} |x_a| < 1 \). It is clear that \( \Sigma(A) \) is a \( \sigma \)-compact subset of \( s(A) \). If \( A = \mathbb{N} \) then \( \Sigma(\mathbb{N}) = \Sigma \) is the radial interior of the Hilbert cube \( Q \).

For \( n \in \mathbb{N} \) and any \( \gamma \in [n, n + 1) \) we denote

\[
C_{\gamma}(\mathbb{I}^n) = \{ f \in C(\mathbb{I}^n) \mid \dim_H(\text{graph} f) > \gamma \}.
\]

Here \( C(\mathbb{I}^n) \) is the space of continuous functions on \( \mathbb{I}^n \) and \( \dim_H \) stands for the Hausdorff dimension.

As usual, by \( \mathbb{Q} \) we denote set of rational numbers. The aim of the talk will be that the system \( \{ C_{\gamma}(\mathbb{I}^n) \}_{n, n + 1} \) is a decreasing \( \mathcal{F}_\sigma \)-absorbing system in \( C(\mathbb{I}^n) \) (in sense of [1]) and there exists a homeomorphism \( \alpha \) from \( C(\mathbb{I}^n) \) onto \( s([n, n + 1) \cap \mathbb{Q}) \) such that for every \( \gamma \in (n, n + 1) \)

\[
\alpha(C_{\gamma}(\mathbb{I}^n)) = \Sigma([n, \gamma) \cap \mathbb{Q}) \times s((\gamma, n + 1) \cap \mathbb{Q}).
\]

This result extends that from [4] where it is proved that for any sequence \( (\gamma_n), n \leq \gamma_1 < \gamma_2 < \ldots < n + 1 \), the sequence of the sets of functions in \( C(\mathbb{I}^n) \) whose graphs are of Hausdorff dimension \( \gamma_n \) forms an \( \mathcal{F}_\sigma \)-absorbing sequence in \( C(\mathbb{I}^n) \).

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**$G_δ$-Points in Compact Spaces**

Henryk Michalewski

*Warsaw University, POLAND, henrykmichalewski@gmail.com*

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**Cardinal Characteristics for Menger-Bounded Subgroups**

Heike Mildenberger

*Universität Wien, Kurt Gödel Research Center for Mathematical Logic, Währinger Str. 25, 1090 Vienna, AUSTRIA, heike@logic.univie.ac.at*

The *Baer-Specker group* is $\mathbb{Z}^\omega$ with pointwise addition. Let $G \subseteq \mathbb{Z}^\omega$ be a subgroup. For $g: \omega \to \mathbb{Z}$, we write $g'(n) = \max\{|g(m)| : m \leq n\}$. Let $k \in \omega \setminus \{0\}$. We say “$G^k$ is Menger-bounded” or “$G$ has Menger-bounded $k$-th power” iff

$$(\exists f \in [\omega]^\omega)(\forall F \subseteq [G]^k)(\exists \infty n)(\forall g \in F)(g'(n) \leq f(n)).$$

This is syntactically the simplest of the equivalent characterisations given in [2, Theorem 5]. Menger-boundedness in a broader sense is defined for topological groups and also called $\sigma$-boundedness. We refer the reader to [3] for more information.

Machura, Shelah and Tsaban showed in [2] that under the condition that a relative $d'(\mathcal{P})$ of the dominating number is at least $\mathfrak{d}$ there are subgroups of the Baer-Specker group whose $k$-th power is Menger-bounded and whose $(k+1)$-st power is not. We show that the sufficient condition implies $\tau \geq \mathfrak{d}$ and indeed can be replaced by $\tau \geq \mathfrak{d}$. This result includes an affirmative answer to a question by Tsaban on a possibly weaker still sufficient condition. We show that it is consistent relative to ZFC that $\mathfrak{g} \leq \tau < \mathfrak{d}$ and there are subgroups of the Baer-Specker group whose $k$-th power is Menger-bounded and whose $(k+1)$-st power is not.

This talk is mainly a report on [3].
The Baire Classification of Partial Derivatives of Functions of Two Variables

Volodymyr Mykhaylyuk

Chernivtsi National University, UKRAINE, mathan@ukr.net

In the talk we shall discuss the Baire classification of partial derivatives of separately differentiable functions.

G. Tolstov [2] proved that the partial derivative $f'_x$ of a separately continuous function $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, which is differentiable with respect to the first variable, is a function of the first Baire class. L. Snyder [1] generalized this result to approximative partial derivatives of functions of two real variables.

**Theorem.** Let $n \in \mathbb{N}$, $X = \mathbb{R}$, $Y$ be a topological space, $f: X \times Y \to \mathbb{R}$ be a function which is continuous with respect to the second variable and such that $E = \{(x,y) \in X \times Y : \text{exists } \frac{\partial f}{\partial y}(x,y)\}$. Then the function $g: E \to \mathbb{R}$, $g(x,y) = \frac{\partial f}{\partial y}(x,y)$ is of the first Baire class.

References


On Two Classes of Functions with the Hahn Property

Vasyl Nesterenko

Chernivtsi National University, UKRAINE, mathan@ukr.net

A function \( f : X \times Y \to Z \) is defined to have the Hahn property if there is a residual subset \( A \) of \( X \) such that \( A \times Y \subseteq C(f) \).

In [1] J. Calbrix and J. Troallic proved that a separately continuous function \( f : X \times Y \to Z \) has the Hahn property if \( X \) is a topological space, \( Y \) is a second countable space and \( Z \) is a metrizable space. In [2] and [3] this result was generalized to other classes of functions, in particular, to the class \( K_hC \) of functions which are horizontally quasicontinuous with respect to the first variable and continuous with respect to the second variable.

Those results suggested to introduce a new class of spaces. A topological space \( Y \) is defined to be a Hahn space if for any topological space \( X \) and any metrizable space \( Z \) a function \( f : X \times Y \to Z \) has the Hahn property provided \( f \) is continuous with respect to the second variable and continuous with respect to the first variable for the values of the second variable that belong to a dense subset of \( Y \). Each second countable space is Hahn, but the converse is not true.

**Theorem 1.** Let \( X \) be a topological space, \( Y \) is a Hahn first countable separable space, \( Z \) is a metrizable space. A function \( f : X \times Y \to Z \) has the Hahn property provided \( f \) is jointly quasicontinuous and almost continuous with respect to the second variable for values of the first variable that belong to some residual set.

Professor O. Skaskiv posed a problem of description of the structure of the set of discontinuity points of separately monotone function \( f : \mathbb{R}^2 \to \mathbb{R} \). It can be proved that such functions are poinwise discontinuous. However the complete description of the set of discontinuity points of such function is still not known. We were able to prove the following theorem.

**Theorem 2.** Let \( Y \) be a separable first countable Hahn space. A function \( f : \mathbb{R} \times Y \to \mathbb{R} \) has the Hahn property provided \( f \) is monotone with respect to the first variable and continuous with respect to the second variable for values of the first variable that belong to some residual subset of \( X \).

References


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**Suslinian Continua**

Jacek Nikiel

*Instytut Matematyki i Informatyki, Uniwersytet Opolski, POLAND*

nikiel@math.uni.opole.pl

A continuum is said to be Suslinian if it contains no uncountable family of nondegenerate and pairwise disjoint subcontinua. A survey of recent results and open problems on Suslinian continua will be given with some indication of methods that are used in studying such continua.

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**Stable Shape: Applications and Open Problems**

Sławomir Nowak

*Institute of Mathematics, University of Warsaw, POLAND*

swnowak@impan.edu.pl

The stable shape category $\text{ShStab}$ of compact spaces is related to stable homotopy as the ordinary shape category $\text{Sh}$ is to homotopy.

The purpose of the talk is to present properties of $\text{ShStab}$. The special emphasis is put on interactions of stable shape with other areas of topology. This approach motivates us to introduce new shape or stable shape invariants and modify the construction of $\text{ShStab}$.

In particular the infinite dimensional stable shape categories of compact spaces are defined. In certain sense they are complementary to $n$-shape category of compact spaces (finite dimensional shape category). The objects of infinite dimensional shape category are isomorphic iff their function spectra have the same $n$-type.

We formulate open problems which concern relationships between shape or stable shape theory and topology or homotopy theory.
Spaces of Capacities on Metric Compacta

Oleh Nykyforchyn

Precarpathian National University, Ivano-Frankivs’k, UKRAINE
nick@pu.if.ua

Given a metric compactum \((X, d)\) let \(M^2 X\) denote the metric space of upper semicontinuous capacities on \(X\) (see [3]) and let \(M^X, M^X\) stand for the subspaces of \(M^2 X\), consisting of necessity measures and all possibility measures on \(X\), see [2].

**Theorem 1.** If \((X, d)\) is an infinite metrizable compactum, then the spaces \(M^X\) and \(M^X\) are homeomorphic to the Hilbert cube.

We denote by \(\mu X : M^2 X \rightarrow M^X\) a component of the multiplication of the capacity monad. Let \(\mu^X : M^2 X \rightarrow M^X\) and \(\mu^X : M^2 X \rightarrow M^X\) be its restrictions.

**Theorem 2.** For each metric compactum \(X\) containing more than one point and each \(\epsilon > 0\) there are continuous mappings \(f_1, f_2 : M^2 X \rightarrow M^2 X\) such that \(\mu X \circ f_1 = \mu X \circ f_2 = \mu X\), \(\text{dist}(f_1, 1_{M^2 X}) < \epsilon, \text{dist}(f_2, 1_{M^2 X}) < \epsilon, f_1(M^2 X) \cap f_2(M^2 X) = \emptyset\).

Thus Fibrewise Disjoint Approximation Property holds for \(\mu X\). In the sequel we regard \(X \subset M^2 X\). Here is a counterpart for possibility measures.

**Theorem 3.** For each metric compactum \(X\) containing more than one point and each \(\epsilon > 0\) there are continuous mappings \(f_1, f_2 : M^2 X \rightarrow M^2 X\) such that \(\mu^X \circ f_1 = \mu^X \circ f_2 = \mu^X\), \(\text{dist}(f_1, 1_{M^2 X}) < \epsilon, \text{dist}(f_2, 1_{M^2 X}) < \epsilon, f_1(M^2 X \setminus X) \cap f_2(M^2 X \setminus X) = \emptyset\).

A similar result holds also for necessity measures.

**References**


Countably Compact Primitive Topological Inverse Semigroups

Kateryna Pavlyk and Oleg Gutik

Department of Algebra,Pidstrygach Institute for Applied Problems of
Mechanics and Mathematics of National Academy of Sciences, Naukova 3b,
Lviv, 79060, UKRAINE, kpavlyk@yahoo.co.uk

We shall follow the terminology of [1, 2, 4]. If $S$ is a semigroup then we
denote the subset of idempotents of $S$ by $E(S)$.

A topological (inverse) semigroup is a topological space together with a
continuous multiplication (and an inversion, respectively).

An inverse semigroup $S$ with zero is called primitive if every idempotent
of $S$ is primitive. A completely 0-simple inverse semigroup is called a Brandt
semigroup.

**Theorem 1.** Any countably compact primitive topological inverse semi-
group $S$ is an orthogonal sum of topological inverse Brandt semigroups with
finite subsemigroups of idempotents. Moreover, $E(S)$ is homeomorphic to
the one-point Alexandrov compactification of infinite discrete space with
zero as a remainder if and only if $E(S)$ is an infinite set.

**Theorem 2.** Any countably compact primitive topological inverse semi-
group is a dense subsemigroup of a compact primitive topological inverse
semigroup.

Let $\mathcal{S}$ be a class of topological semigroups.

A semigroup $S \in \mathcal{S}$ is called H-closed in $\mathcal{S}$, if $S$ is a closed subsemigroup
of any topological semigroup $T \in \mathcal{S}$ which contains $S$ as subsemigroup. If
$\mathcal{S}$ coincides with the class of all topological semigroups, then the semigroup
$S$ is called H-closed. H-closed topological semigroups were introduced by
J. W. Stepp in [5], and there they were called maximal semigroups.

A topological semigroup $S \in \mathcal{S}$ is called absolutely H-closed in the class
$\mathcal{S}$, if any continuous homomorphic image of $S$ into $T \in \mathcal{S}$ is H-closed in
$S$ [3, 4].

**Theorem 3.** A primitive topological inverse semigroup $S$ is (absolutely)
H-closed in the class of topological inverse semigroups if and only if any
maximal subgroup of $S$ is an (absolutely) H-closed semigroup in the class
of topological inverse semigroups.
References


Decompositions of Topological Spaces and Total Recurrence

Oleksandr Petrenko and I. V. Protasov

Department of Cybernetics, Kyiv National University, UKRAINE

opetrenko72@gmail.com

We say that a subspace $A$ of a topological space $X$ is *almost discrete* if, for every $a \in A$, there exists a neighbourhood $U$ of $a$ in $X$ such that $A \cap U$ is finite. If $X$ is a $T_1$-space then every almost discrete subspace of $X$ is discrete.

We say that an infinite topological space $X$ is *cofinite* if each proper closed subspace of $X$ is finite. Given an infinite set $X$, there exists only one cofinite $T_1$-topology $\tau$ on $X$: $U \in \tau$ if and only if either $U = \emptyset$ or $X \setminus U$ is finite.

**Theorem 1.** Every topological space $X$ can be partitioned $X = F \cup AD \cup CF$ where $F$ is finite, $AD$ is a disjoint union of countable almost discrete subspaces, $CF$ is a disjoint union of cofinite subspaces.

We say that a subspace $A$ of a topological space $X$ is *Hausdorff* if, for any $a, b \in A$, there exist neighbourhoods $U, V$ of $a, b$ in $X$ such that $U \cap V = \emptyset$. 
We say that a subspace $A$ of a topological space $X$ is linked if, for any $a, b \in A$ and any neighbourhoods $U, V$ of $a, b$ in $X$ one has $U \cap V \neq \emptyset$.

**Theorem 2.** Every topological space $X$ can be partitioned, $X = F \cup H \cup L$, where $F$ is finite, $H$ is a disjoint union of infinite Hausdorff subspaces, $L$ is a disjoint union of infinite linked subspaces.

By [2], every infinite Hausdorff space is either a one-point compactification of an infinite discrete space, or can be partitioned in countable discrete subspaces.

**Theorem 3.** Let $X$ be an infinite topological space without cofinite subspaces. Then one of the following statements holds

(i) $X$ is a disjoint union of countable almost discrete subspaces;

(ii) there exists a non-empty finite subspace $F \subset X$ such that, for every $a \in F$ and every neighbourhood $U$ of $a$, $X \setminus U$ is finite.

A topological space $X$ is called totally recurrent [1] (resp. bijectively recurrent [2]) if every, not necessarily continuous, mapping (resp. bijection) $f : X \to X$ has a recurrent point. Recall that $x \in X$ is recurrent if $x$ is a limit point of the orbit $(f^n(x))_{n \in \omega}$. By [2], an infinite Hausdorff space $X$ is totally (bijectively) recurrent if and only if $X$ is a one-point compactification of an infinite discrete space.

**Theorem 4.** Let $X$ be an infinite topological space without cofinite subspaces. Then one of the following statements are equivalent

(i) $X$ is totally recurrent;

(ii) $X$ is bijectively recurrent;

(iii) there exists a non-empty finite subspace $F \subset X$ such that, for every $a \in F$ and every neighbourhood $U$ of $a$, $X \setminus U$ is finite.

**Theorem 5.** If a topological space $X$ has no infinite almost discrete subspaces (in particular, $X$ is cofinite), then $X$ is totally recurrent.

**References**


Some Applications of the Kuratowski-Ulam Theorem

Szymon Plewik

Institute of Mathematics, University of Silesia, Katowice, POLAND
plewik@math.us.edu.pl

Balleans of Bounded Geometry and G-Spaces

I. V. Protasov

Department of Cybernetics, Kyiv National University, UKRAINE
protasov@unicyb.kiev.ua

A ball structure is a triplet $B = (X, P, B)$, where $X, P$ are non-empty sets and, for any $x \in X$ and $\alpha \in P$, $B(x, \alpha)$ is a subset of $X$ which is called a ball of radius $\alpha$ around $x$. It is supposed that $x \in B(x, \alpha)$ for all $x \in X$, $\alpha \in P$. The set $X$ is called the support of $B$, $P$ is called the set of radii. Given any $x \in X$ and $\alpha \in P$, we put $B^*(x, \alpha) = \{y \in X : x \in B(y, \alpha)\}$.

A ball structure $X$ is called a ballean if

- for any $\alpha, \beta \in P$, there exist $\alpha', \beta' \in P$ such that, for every $x \in X$,
  
  $B(x, \alpha) \subseteq B^*(x, \alpha'), \quad B^*(x, \beta) \subseteq B(x, \beta');$

- for any $\alpha, \beta \in P$, there exists $\gamma \in P$ such that, for every $x \in X$,
  
  $B(B(x, \alpha), \beta) \subseteq B(x, \gamma).$

Let $B_1 = (X_1, P_1, B_1)$, $B_2 = (X_2, P_2, B_2)$ be balleans. A mapping $f: X_1 \to X_2$ is called a $<\text{-}mapping$ if, for every $\alpha \in P_1$, there exists $\beta \in P_2$ such that, for every $x \in X_1$,

$f(B_1(x, \alpha)) \subseteq B_2(f(x), \beta).$

The category of balleans and $<\text{-}mappings$ can be considered as an asymptotic reflection of the category of uniform spaces and uniformly continuous mappings (see [1], [2]).
Let $B = (X, P, B)$ be a ballean, $\alpha \in P$. A subset $S$ of $X$ is called $\alpha$-separated if $B(x, \alpha)$ and $B(y, \alpha)$ are disjoint for all distinct $x, y \in S$. An $\alpha$-capacity of a subset $Y$ of $X$ is the cardinal

$$cap_\alpha = \sup\{|S| : S \text{ is an } \alpha - \text{separated subset of } Y\}.$$ 

We say that a ballean $B = (X, P, B)$ has bounded geometry if there exist $\beta \in P$ and a function $h : P \to \omega$ such that $cap_\beta(B(x, \alpha)) \leq h(\alpha)$ for all $x \in X$, $\alpha \in P$.

Let $G$ be a group, $X$ be a $G$-space. We denote by $\mathcal{F}_G$ the family of all finite subset of $G$ containing the identity of $G$, and get the ballean $B(G, X) = (X, \mathcal{F}_G, B)$, where $B(x, F) = \{f : f \in F\}$ for all $x \in X$, $F \in \mathcal{F}_G$. Clearly, $B(G, X)$ is of bounded geometry.

Let $S$ be a set, $B = (X, P, B)$ be a ballean, $f, g : S \to X$. We say that $f, g$ are close if there exists $\alpha \in X$ such that $f(x) \in B(g(x), \alpha)$ for every $x \in X$. Two baleans $B_1, B_2$ with the supports $X_1, X_2$ are called coarsely equivalent if there exist the $<$-mappings $f_1 : X_1 \to X_2$ and $f_2 : X_2 \to X_1$ such that $f_1 \circ f_2$ and $f_2 \circ f_1$ are close to corresponding identity mappings of $X_1$ and $X_2$.

**Theorem 1.** Every ballean of bounded geometry is coarsely equivalent to a ballean $B(G, B)$ of some $G$-space $X$.

**References**


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**Kaleidoscopolical and Hamming Graphs**

Ksenia Protasova

Department of Cybernetics, Kyiv National University, UKRAINE
islab@unicyb.kiev.ua

Let $\Gamma(V, E)$ be a connected graph with the set of vertices $V$ and the set of edges $E$. Given $u, v \in V$ and $r \in \mathbb{N}$, we denote by $d(u, v)$ the length of a shortest path between $u$ and $v$, and put $B(v, r) = \{x \in V : d(x, v) \leq r\}$.
Let \( s > 1 \) be a natural number. A graph \( \Gamma(V, E) \) is said to be regular of degree \( s \) if \( |B(v, 1)| = s + 1 \) for every \( v \in V \).

A regular graph \( \Gamma(V, E) \) of degree \( s \) is called kaleidoscopical if there exists a coloring \( \chi : V \rightarrow \{0, 1, \ldots, s\} \) such that \( \chi \) is a bijection on every ball \( B(v, 1), v \in V \).

Let \( G \) be a group with the identity \( e \) and let \( X, Y \) be subsets of \( G \). The following definition is a correction of corresponding definition from [1, p. 60].

**Definition.** We say that \((X, Y)\) is a kaleidoscopical pair in \( G \) provided that \( X \) is finite and the following conditions hold:

(i) \( e \in X, X = X^{-1} \) and \( G = \langle X \rangle \), where \( \langle X \rangle \) is the subgroup generated by \( X \);

(ii) \( e \in Y, G = XY \);

(iii) \( XX \cap Y^{-1}Y = X \cap YY^{-1} = \{e\} \).

By this definition, every element \( g \in G \) has a unique representation \( g = xy, x \in X, y \in Y \). We define the standard coloring \( \chi : G \rightarrow X \) by the rule \( \chi(g) = x \). We remind that the Cayley graph \( \text{Cay}(G, X) \) is the graph with the set of vertices \( G \) and the set of edges \( \{(x, y) : x, y \in G, x \neq y, xy^{-1} \in X \} \).

**Theorem 1.** Let \( G \) be a group with kaleidoscopical pair \((X, Y)\). Then \( \text{Cay}(G, X) \) with standard coloring \( \chi \) is a kaleidoscopical graph.

A kaleidoscopical pair \((X, Y)\) is called a Hamming pair if \( Y \) is a subgroup of \( X \). In this case, the kaleidoscopical graph \( \text{Cay}(G, X) \) is called a Hamming graph.

**Theorem 2.** Let \((X, Y)\) be a kaleidoscopical pair in a group \( G \), \( \chi \) be the standard coloring of \( G \). Then the following statements are equivalent

(i) \((X, Y)\) is a Hamming pair;

(ii) \( g_1, g_2 \in G, x \in X \) and \( \chi(g_1) = \chi(g_2) \) then \( \chi(x.g_1) = \chi(x.g_2) \).

Let \((X, Y)\) be a Hamming pair in a group \( G \). Since \( X \) is finite, \( G \) is finitely generated. Since \( G = XY \), we have that \( Y \) is a subgroup of finite index. Every subgroup of finite index of a finitely generated group is also finitely generated, so \( Y \) is finitely generated.

**Theorem 3.** For every finitely generated group \( Y \), there exists a group \( G \) and a finite subset \( X \subseteq G \) such that \( Y \) is a subgroup of \( G \) and \((X, Y)\) is a Hamming pair in \( G \).
References

Towards the History of Topology in Lviv

Yaroslav Prytula

*Ivan Franko National University of Lviv, UKRAINE*

For the first time, topology appeared in the curricula at Lviv University in 1898/99 academic year. That year Prof. J. Puzyna announced the course "Topological Studies" which contained initial notions of the set theory and topology described in his monograph "Teoria funkcji analitycznych" (Lwów, 1898-1900; in Polish). It was W. Sierpiński who initiated research in topology in 1908. S. Mazurkiewicz was granted PhD in 1913 for his thesis "Przyczynki do teorii mnogości" under supervision J. Puzyna. Z. Janiszewski delivered the lectures during the period 1913-18 (PhD for the thesis "Sur les continus irréductibles entre deux points" under the supervision of H. Lebesgue). In 1918 W. Sierpiński and Z. Janiszewski moved to Warsaw as they received professorships at the Warsaw University.

During 20-30s the Lviv mathematicians (S. Banach, J. Schauder, S. Mazur e. a.) studied topological properties of Banach spaces as also applications of topological methods, in particular, in the theory of partial differential equations.

In 1927-33 K. Kuratowski was the head of the III chair of mathematics at the Lviv Polytechnica. In this time he wrote his monograph in topology. S. Ulam was one of his disciples.

In 1940-41 B. Knaster, E. Marczewski (Spilrajn), S. Saks, M. Wodzicki taught at the Lviv University.

During 20-30s, outstanding mathematicians (J. von Neumann, H. Lebesgue, E. Zermelo, P. Aleksandrov, S. Sobolev e. a.) visited Lviv.
Extension of the Mappings to the Topological Groups

N. M. Pyrch

Ukrainian Academy of Printing, Lviv, UKRAINE, pnazar@ukr.net

A subspace $Y$ of a topological space $X$ is called a $G$-retract of $X$, if any continuous mapping from $Y$ to a Hausdorff topological group $G$ admits a continuous extension on $X$.

**Proposition 1.** Let $X$ be a topological space, $r_1, r_2$ its retractions, such that $r_1 \circ r_2(X) = r_2 \circ r_1(X)$. Then the subspace $K = r_1(X) \cup r_2(X)$ is a $G$-retract of a topological space $X$.

**Proposition 2.** Let $Y$ be a subspace of a functionally Hausdorff space $X$. Let $(F(X), \eta_X)$, $(F(Y), \eta_Y)$ be the free topological groups on $X$ and $Y$ (in the sense of [1]). Then the following are equivalent:

1) $Y$ is a $G$-retract in $X$;

2) there exists a homomorphism $h: F(X) \to F(Y)$ such that $h(\eta_X(y)) = \eta_Y(y)$ for all $y \in Y$.

It follows from this proposition that any $G$-retract of a functionally Hausdorff topological space $X$ is closed in $X$.

A topological space $X$ is called an absolute $G$-retract, if $X$ is a $G$-retract of any topological space $Y$ containing $X$ as a closed subspace.

**Proposition 3.** Let $X$ be a topological space $A$ its closed subspace such that topological spaces $A$ and $X/A$ are absolute $G$-retracts. Then the topological space $X$ is an absolute $G$-retract.

References

Transfinite Extension of Asymptotic Dimension

Taras Radul

Ivan Franko National University of Lviv, UKRAINE
tarasradul@yahoo.co.uk

Asymptotic dimension asdim of a metric space was defined by M. Gromov for studying asymptotic invariants of discrete groups, particularly fundamental groups of manifolds [1]. Then G. Yu found a successful application of the asymptotic dimension to prove a series of conjectures, including the famous Novikov Higher Signature conjecture, for groups with finite asdim [2]. Many authors started to develop a general theory called asymptotic topology or large-scale geometry (see for example the survey [3]). Asymptotic topology studies global properties of (unbounded) metric spaces neglecting small (bounded) details of these spaces. Thus, the properties and invariants of metric spaces under consideration are treated in the limit, at infinity. The large-scale world, the macrocosm, admits an analogue with the microcosm, where the limit is taken at a point. A significant part of topology is devoted to the investigation of local properties of spaces and can thus be referred to the microcosm. Dimension theory can serve as example. It became important to carry over topological theories to macrocosm.

The dimension asdim can be considered as asymptotic analogue of the Lebesgue covering dimension dim. Dranishnikov has introduced dimensions asInd and asind which are analogous to large inductive dimension Ind and small inductive dimension ind [4, 5]. There many analogues and differences between topological and asymptotic dimension theories. We discuss basic theorems of dimension theory like sum theorem, addition theorem and subspace theorem.

M. Zarichnyi has proposed to consider transfinite extension of asInd and conjectured that this extension is trivial. This conjecture is proved in [6]: if a space has a transfinite asymptotic dimension, then its dimension is finite. We obtain the same result for asind.

We define as well a transfinite extension tradim and show that this extension is not trivial: there exist metric spaces with transfinite infinite dimension.

References

Bernstein Sets and k-Coverings

Robert Rałowski, Jan Kraszewski, Przemysław Szczepaniak, and Szymon Źeberski

Institute of Mathematics and Computer Science, Wroclaw University of Technology, POLAND, Robert.Ralowski@pwr.wroc.pl

We study a notion of $k$-covering in connection with Bernstein sets and other version of nonmeasurability. Our results corresponds to Muthuvel [1] and Nowik see [2]. We consider various versions of $k$-coverings.

References


The Continuity of the Inverse in Paratopological Groups, the Pseudocompactness and the Periodicity

Alex Ravsky

Pidstrygach Institute for Applied Problems of Mechanics and Mathematics of National Academy of Sciences, Naukova 3b, Lviv, 79060, UKRAINE
oravsky@mail.ru

Under a paratopological group we understand a pair \((G, \tau)\) consisting of a group \(G\) and a topology \(\tau\) on \(G\) making the group operation \(\cdot : G \times G \to G\) of \(G\) continuous. If, in addition, the operation \((\cdot)^{-1} : G \to G\) of taking the inverse is continuous with respect to the topology \(\tau\), then \((G, \tau)\) is a topological group. A topological space is pseudocompact if every locally finite family of open subsets of the space is finite. A Tychonoff space is pseudocompact if and only if every real-valued continuous function on the space is bounded. Every countably compact space is pseudocompact. A topological space is quasiregular if every nonempty open set contains the closure of some nonempty open set. A paratopological group \(G\) is saturated if, for any neighborhood \(U \subseteq G\) of the unit, the set \(U^{-1}\) has nonempty interior in \(G\). A paratopological group \(G\) is a topologically periodic if for every neighborhood \(U\) of the unit and every element \(x \in G\) there exists a number \(n > 0\) such that \(x^n \in U\). A paratopological group \(G\) is 2-pseudocompact if, for every decreasing sequence \(\{U_n\}\) of non-empty open subsets of \(G\), the intersection of the closures of \(U_n^{-1}\) is not empty. Clearly, each countably compact paratopological group is 2-pseudocompact. A paratopological group is \(G\) left \(\omega\)-precompact if for each neighborhood \(U\) of the unit there is a countable subset \(F\) of \(G\) such that \(FU = G\).

**Proposition 1.** For a paratopological group \(G\) the following are equivalent:

1) \(G\) is pseudocompact quasiregular;
2) \(G\) is pseudocompact topologically periodic Baire;
3) \(G\) is 2-pseudocompact saturated;
4) \(G\) is 2-pseudocompact topologically periodic;
5) \(G\) is 2-pseudocompact left \(\omega\)-precompact;
6) \(G\) is a pseudocompact topological group.

**Proposition 2.** Every Baire periodic paratopological group is a topological group.
On a Topological Semigroup Finite Transformations of a Hausdorff Topological Space

Andriy Reiter and Oleg Gutik

Ivan Franko National University of Lviv, UKRAINE
reiter.andriy@yahoo.com

We shall follow the terminology of [1].

A topological (inverse) semigroup is a topological space together with a continuous multiplication (and an inversion, respectively). If $S$ is a semigroup and $\tau$ is a topology on $S$ such that $(S, \tau)$ is a topological semigroup, then we shall call $\tau$ a semigroup topology on $S$.

Let $X$ be a Hausdorff topological space. Let $I_{\text{fin}}(X)$ denote the set of all partial one-to-one finite transformations of $X$ together with the following semigroup operation:

$$x(\alpha \beta) = (xa)\beta \text{ if } x \in \text{dom}(\alpha \beta) = \{ y \in \text{dom} \alpha \mid y \alpha \in \text{dom} \beta \},$$

for $\alpha, \beta \in I_{\text{fin}}$. The semigroup $I_{\text{fin}}(X)$ is called the finite symmetric inverse semigroup over the topological space $X$.

**Definition 1** [3, 4]. Let $\mathcal{S}$ be a class of topological semigroups. A semigroup $S$ is called algebraically $h$-closed in the class $\mathcal{S}$ if for any topology $\tau$ on $S$ such that $(S, \tau) \in \mathcal{S}$ and for any topological semigroups $T \in \mathcal{S}$ and any continuous homomorphism $h : S \to T$ we have that $h(S)$ is a closed subsemigroup of $T$.
Theorem 1. For any Hausdorff topological space the semigroup \( I_{\text{fin}}(X) \) is algebraically h-closed in the class of topological inverse semigroups.

Definition 2 [2]. A Bohr compactification of a topological semigroup \( S \) is a pair \((\beta, B(S))\) such that \( B(S) \) is a compact topological semigroup, \( \beta: S \to B(S) \) is a continuous homomorphism, and if \( g: S \to T \) is a continuous homomorphism of \( S \) into a compact semigroup \( T \), then there exists a unique continuous homomorphism \( f: B(S) \to T \) such that the diagram

\[
\begin{array}{ccc}
S & \xrightarrow{\beta} & B(S) \\
\downarrow{g} & & \downarrow{f} \\
T & & \\
\end{array}
\]

commutes.

Theorem 2. Let \( X \) be an infinite Hausdorff topological space and \( \tau \) be a Hausdorff semigroup topology on \( I_{\text{fin}}(X) \). Then the Bohr compactification of \( (I_{\text{fin}}(X), \tau) \) is a trivial semigroup.

References


On Connected Partitions in Cones

Miroslawa Reńska

Instytut Matematyki, Uniwersytet Warszawski, POLAND
mrenskajimuw.edu.pl

We show that in the cylinder $X \times I$ over a non-locally connected metrizable continuum there exists a partition $L$ between the top and the bottom of the cylinder such that $L$ does not contain any connected partition between these sets. Therefore a metrizable continuum $Y$ is locally connected if and only if every partition in the cylinder over $Y$ between the bottom and the top of the cylinder contains a connected partition between these sets. In addition, we show that our arguments extend to the case of some non-metrizable non-locally connected continua. J. Krasinkiewicz asked whether for every metrizable continuum $X$ there exists a partition $L$ between the top and the bottom of the cylinder $X \times I$ such that $L$ is a hereditarily indecomposable continuum. The theorem above shows that in the case of non-locally connected continua we can not construct such a partition just repeating the construction of Bing. We present a construction of such partitions for some non-locally connected continua of simple form.

The Bing-Borsuk and the Busemann Conjectures

Dušan Repovš

University of Ljubljana, SLOVENIA, dusan.repovs@guest.arnes.si

We shall present a survey of two classical conjectures concerning the characterization of manifolds: the Bing-Borsuk Conjecture asserts that every $n$-dimensional homogeneous ANR is a topological $n$-manifold, whereas the Busemann Conjecture asserts that every $n$-dimensional $G$-space is a topological $n$-manifold.

The key objects in both cases are so-called generalized manifolds, i.e. ENR homology manifolds. We shall look at their history, from the early beginnings in 1930’s to the present day, concentrating on those geometric properties of these spaces which are particular for dimensions 3 and 4, in comparison with generalized $(n > 4)$-manifolds.
In the second part of the talk we shall present the current state of the two conjectures (the work of Bing-Borsuk, Bestvina-Daverman-Venema-Walsh, Brahms, Bryant-Ferry-Mio-Weinberger, Busemann, Cannon, Daverman-Re-
upakan-Thickstun, Halverson-Repovš, Edwards, Krakus, Lacher-
upakan-Thickstun, Pedersen-Quinn-Ranicki, Thurston, and others). We shall also list
open problems and related conjectures.

The Hyperspace of Indecomposable
Subcontinua of a Cube

Alicja Samulewicz

Silesian University of Technology, POLAND, Alicja.Samulewicz@polsl.pl

The hyperspace of all indecomposable continua in the cube of dimension
greater than 2 is homeomorphic to the separable Hilbert space $l_2$.

Plausibility Measures on Ultrametric Spaces

Alexander Savchenko

Kherson State Agrarian University, UKRAINE, Savchenko1960@rambler.ru

Let $P$ denote a partially ordered set (its partial order is denoted by
$\leq$) for which there exist the minimal element $\bot$ and the maximal element
$\top \neq \bot$.

Let $\mathcal{B}$ the $\sigma$-algebra of Borel subsets of a space $X$. A plausibility measure
on $X$ is a map $\mu: \mathcal{B} \to P$ satisfying the following properties: (1) $\mu(\emptyset) = \bot$;
(2) $\mu(X) = \top$; (3) $A \subset B$, $A, B \in \mathcal{B}$, then $\mu(A) \leq \mu(B)$.

The following is an example of plausibility measure. Let $M$ be a finite
nonempty subset of $X$. We consider the power-set $2^M$ partially ordered by
inclusion. We define the plausibility measure $\mu_M$ by the formula $\mu_M(A) =
M \cap A$. More generally, every map $f: M \to X$ determines a plausibility
measure $\mu_f$ defined by: $\mu_f(A) = f^{-1}(A)$.

We fix $P$ and denote by $\text{Pl}_P(X)$ the set of all plausibility measures on $X$ with
finite supports on $X$ (we say that the support of $P$ is finite (compact)
if there exists a finite (compact) subset $A$ of $X$ such that $\mu(Y) = \mu(A)$, for
any $Y \in \mathcal{B}$).
Let \( (X, d) \) be an ultrametric space. For any \( \varepsilon > 0 \), let \( O_\varepsilon \) denote the family of subsets of \( X \) that can be represented as unions of (open) balls of radius \( \varepsilon \). We let

\[
\hat{d}(\mu, \nu) = \inf \{ \varepsilon > 0 \mid \mu(U) = \nu(U) \text{ for any } U \in O_\varepsilon \}.
\]

Then \( \hat{d} \) is an ultrametric on the set \( \text{Pl}_f(X) \). The elements of the completion of \( (\text{Pl}_f(X), \hat{d}) \) are the plausibility measures with compact supports on \( X \).

We investigate the obtained functor on the category of ultrametric spaces and nonexpanding maps.

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**Infinite-Dimensional Manifolds Related to \( C \)-Spaces**

O. Shabat

*Ukrainian Academy of Printing, Lviv, UKRAINE*

A topological space \( X \) has property \( C \) (briefly is a \( C \)-space) if for each sequence \( \{\alpha_n \mid n \in \mathbb{N}\} \) of open coverings of \( X \) there exists a sequence \( \{\beta_n \mid n \in \mathbb{N}\} \) of open disjoint families such that each family \( \beta_n \) refines \( \alpha_n \) and the family \( \cup_{n=1}^{\infty} \beta_n \) is a cover of \( X \) (see [1]). The transfinite dimension \( \dim_C \) is introduced by Borst [2]. It is proved in [2] that a compact metrizable space \( X \) is a \( C \)-space if and only if \( \dim_C(X) < \omega_1 \).

T. Radul [3] proved that for every countable ordinal \( \alpha \) there exists a compact metrizable \( C \)-space \( X \) which contains a topological copy of every compact metrizable space \( Y \) with \( \dim_C(Y) \leq \alpha \).

This result allows us to construct, for a cofinal subset \( A \) of \( \omega_1 \), and every \( \alpha \in A \) a \( k_\omega \)-space \( O_\alpha \) which can be considered as a model space of the theory of infinite-dimensional manifolds. In the class of compact metrizable spaces \( X \) with \( \dim_C(Y) < \alpha \), the \( O_\alpha \)-manifolds play the role corresponding to that of \( \mathbb{R}^\infty \)-manifolds in the class of finite-dimensional compact metrizable spaces.

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Morse Numbers of Flows with Toroidal Non-Wandering Set

Volodymyr Sharko

Institute of Mathematics, NAS of Ukraine, Tereshchenkivs’ka str., 3, Kyiv, 01601, UKRAINE, sharko@ukrpack.net

A flow $\varphi_t$ on smooth closed manifold $M^n$ belong to the class $\Gamma(T^k)$ if the set of non-wandering points $\Omega(\varphi_t)$ of $\varphi_t$ consists of a disconnected union of embedded $k$-tori ($2 \leq k \leq n - 2$) which have normal hyperbolic structure. Any $k$-tori from flow $\varphi_t \in \Gamma(T^k)$ have index $i$ ($0 \leq i \leq n - k$).

By definition the $i$-th Morse $T^k$-number of a manifold $M^n$, $\mathcal{M}^{T^k}_i(M^n)$ is the minimum number of $k$-tori of the index $i$ taken over all flows from the class $\Gamma(T^k)$ on $M^n$.

Let $M^n$ ($n > k + 6$) be a arbitrary closed smooth manifold with zero Euler characteristic. Then:

1) for manifold $M^n$ the class $\Gamma(T^k)$ is non-empty;
2) $\mathcal{M}^{T^k}_0(M^n) = 1$;
3) $\mathcal{M}^{T^k}_{n-k}(M^n) = 1$;
4) $\mathcal{M}^{T^k}_i(M^n) = 0$ for $k + 1 \leq i \leq n - k - 1$.

Let $M^n$ be a closed smooth manifold with zero Euler characteristic, by definition the $T^k$-Morse number of the manifold $M^n$, $\mathcal{M}^{T^k}(M^n)$ is the minimum number of $k$-tori of all indices taken over all flows from the class $\Gamma(T^k)$ on $M^n$.

We give estimate for the value of $T^k$-Morse number of the manifold $M^n$ ($n > k + 6$) in the terms of homology groups of $M^n$.
Abstracts of Reports

Absolute Extensors and Functors in the Asymptotic Category

O. Shukel

Ivan Franko National University of Lviv, UKRAINE, oshukel@gmail.com

The asymptotic category \( \mathcal{A} \) is defined as follows [2]. The objects of \( \mathcal{A} \) are proper metric spaces and the morphisms are the proper asymptotically Lipschitz maps. Recall that a map \( f: (X, d) \mapsto (Y, \rho) \) of metric spaces is called asymptotically Lipschitz if there exist \( \lambda > 0, s \geq 0 \) such that \( \rho(f(x), f(y)) \leq \lambda d(x, y) + s \) for every \( x, y \in X \).

Given two proper metric spaces, \( X \) and \( Y \), the Kuratowski symbol \( X \tau Y \) means that every proper asymptotically Lipschitz map \( f: A \to Y \) defined on a closed subset \( A \) of \( X \) admits an extension over \( X \) which is also an asymptotically Lipschitz map.

We say that a proper metric space \( X \) is an absolute extensors for a class \( \mathcal{C} \) of proper metric spaces of \( Y \tau X \), for every \( Y \in \mathcal{C} \). If \( \mathcal{C} \) coincides with the class of all proper metric spaces (of the asymptotic dimension \( \leq n \); see [3]), we say that \( X \) is an absolute extensor (in asymptotic dimension \( n \)).

The \( n \)-th hypersymmetric power functor \( \exp_n \) and \( G \)-symmetric power functor \( SP^n_0 \) are examples of normal functors in the asymptotic category \( \mathcal{A} \) (see [4]).

The main results of the talk concern preservation of the classes of absolute extensors (in asymptotic dimension \( n \)) by the mentioned \( G \)-symmetric power functors and the \( n \)-th hypersymmetric power functors.

These results are counterparts of the corresponding results of Basmanov on preservation of absolute extensors by covariant functors of finite degree in the category of compact Hausdorff spaces [1].

References

Extensors of Spaces with Filtration

Zoya Silaeva

Belarus State University, Minsk, BIELORUSSIA, szn2006@yandex.ru

The investigation of spaces with filtrations takes important place and plays important role in many branches of mathematics, in particular in homotopy topology. However, the problem of extensions of filtered maps is still at the initial stage. For example, the question of the identity of absolute extensors and absolute retracts in this category is unanswered. The same is in relation to the question of the role of connectivity properties of filtered elements for the possibility of the filtered maps to be extended. In the talk we intend to give results, answering these and some other questions.

Unconditionally Closed and Algebraic Sets in Groups

Olga Sipacheva

Department of General Topology and Geometry, Mechanics and Mathematics
Faculty, Moscow State University, RUSSIA, o-sipa@yandex.ru

A subset $A$ of a group $G$ is said to be unconditionally closed in $G$ if it is closed in any Hausdorff group topology on $G$ (A. A. Markov, 1945). Note that a group $G$ is non-topologizable (i.e., the only Hausdorff group topology which $G$ admits is discrete) if and only if the complement of the identity element is unconditionally closed in $G$.

Clearly, all solution sets of equations in $G$, as well as their finite unions and arbitrary intersections, are unconditionally closed. Such sets are called algebraic. Markov’s precise definition is as follows.

A subset $A$ of a group $G$ with identity element 1 is said to be elementary algebraic in $G$ if there exists a word $w = w(x)$ in the alphabet $G \cup \{x^\pm 1\}$ ($x$ is a variable) such that

$$A = \{x \in G : w(x) = 1\}.$$ 

Finite unions of elementary algebraic sets are called additively algebraic sets. An arbitrary intersection of additively algebraic sets is said to be
algebraic. Thus, the algebraic sets in $G$ are the solution sets of arbitrary conjunctions of finite disjunctions of equations.

In 1945, Markov showed that any algebraic set is unconditionally closed and posed the problem of whether the converse is true. Moreover, he proved that any unconditionally closed set in a countable group is algebraic. In 1979, Hesse constructed an example in his PhD dissertation showing that the answer to the general question is no. We show that Shelah’s CH example of a non-topologizable group, which has amazing additional properties, also provides an example.

An obvious necessary condition for the topologizability of a group $G$ is that no finite system of inequations in $G$ can have a unique solution (i.e., the complement to an additively algebraic set cannot be one-point). In 1977, Podewski suggested a natural sufficient condition that no system of fewer than $|G|$ inequations has a unique solution; he called groups with this property ungebunden.

We discuss sufficient conditions for the coincidence of unconditionally closed and algebraic sets. We also introduce families of unconditionally $\tau$-closed and $\tau$-algebraic sets in a group, which are natural generalizations of unconditionally closed and algebraic sets defined by Markov, and find a sufficient condition for the coincidence of these families. In particular, we show that these families coincide in any group of cardinality at most $\tau$. This result generalizes both Markov’s theorem on the coincidence of unconditionally closed and algebraic sets in a countable group and Podewski’s theorem on the topologizability of any ungebunden group.

We also touch upon the relationship between the notion of hereditary topologizability of groups (introduced recently by Gabor Lukacs) and that of categorical compactness, or $c$-compactness of groups (introduced by Dikranjan and Uspenskij), which was first noticed by Lukacs.

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**A Continuous Extension Operator for Convex Metrics**

**I. Stasyuk** and **E. D. Tymchatyn**

*Lviv National University, UKRAINE and University of Saskatchewan, CANADA, i_stasyuk@yahoo.com*

The problem of extension of metrics has a long history. Various results on extensions of (pseudo)metrics (which in particular satisfy special
properties) were obtained by many authors (see for instance [1, 2, 5, 6]). We consider the problem of simultaneous extension of continuous convex metrics defined on subcontinua of a Peano continuum.

A metric $r$ on a Peano continuum $X$ is said to be convex if for each $x, y \in X$ there is an arc $[xy]$ with endpoints $x$ and $y$ such that $[xy]$ is isometric to the closed interval $[0, r(x, y)]$ in the real line $\mathbb{R}$. It is known [3] and [4] that a metric continuum is locally connected if and only if it has a convex metric.

If $X$ is a Peano continuum and $A$ is a locally connected subcontinuum of $X$ let $\mathcal{CM}(A)$ denote the set of continuous convex metrics on $A$. We identify each metric $\rho \in \mathcal{CM}(A)$ with its graph which is a compact subset of the space $X \times X \times \mathbb{R}$. Let

$$\mathcal{CM} = \bigcup \{\mathcal{CM}(A) : A \text{ is a Peano subcontinuum of } X\}.$$

We assume that the distance between two metrics is the Hausdorff distance between their graphs. It follows from Bing [2] that for every convex metric on a subcontinuum of a Peano continuum there exists a convex extension onto the whole space. We modify Bing’s construction in order to obtain a continuous simultaneous extension operator. The following theorem is our main result.

**Theorem.** Let $X$ be a Peano continuum. There is a continuous extension operator $u : \mathcal{CM} \to \mathcal{CM}(X)$.

**References**


Continuously Homogeneous Spaces and Topological Right Loops

Sławomir Turek, Taras Banakh and Zdisław Kosztołowicz

Uniwersytet Humanistyczno-Przyrodniczy im. Jana Kochanowskiego w Kielcach, POLAND, slawomir.turek@pu.kielce.pl

A topological space $X$ is defined to be continuously homogeneous if for any two points $x, y \in X$ we can choose a homeomorphism $h_{x,y} : X \to X$ so that $h_{x,y}(y) = x$ and $h_{x,y}$ continuously depends on the points $x, y$ in the sense that the map

$$H : X \times X \times X \to X \times X \times X, \quad H : (x, y, z) \mapsto (x, y, h_{x,y}(z))$$

is a homeomorphism. We shall prove that a topological space $X$ is continuously homogeneous if and only if $X$ admits a continuous binary operation $*: X \times X \to X$ turing $X$ into a topological right loop. The latter means that $X$ has a unit (i.e., an element $\theta \in X$ such that $x \ast \theta = \theta \ast x = x$ for all $x \in X$) and for any $a, b \in X$ the equation $a \ast x = b$ has a unique solution $x = a \setminus b$ that continuously depend on $a, b$. By their topological properties continuously homogeneous spaces resemble topological groups so that many results well-known for topological groups still remain valid for continuously homogeneous spaces. Each infinite zero-dimensional continuously homogeneous space is homeomorphic to the Cantor cube $\{0, 1\}^{w(X)}$ (which carries the structure of a topological group). On the other hand, the 7-dimensional sphere $S^7$ is continuously homogeneous but is not homeomorphic to a topological group. Also there are zero-dimensional continuously homogeneous spaces that are not homeomorphic to topological groups (and even to topological loops).
Topological Proof of the Existence of Market Equilibrium

Marian Turzański

Katowice and Warszawa, POLAND, mturz@ux2.math.us.edu.pl

The traditional model of market equilibrium supports impressive existence results, including the celebrated Arrow-Debreu Theorem. In the talk we shall use topological arguments to determine equilibrium points. General equilibrium theory has the status of the top in mathematical economics. However except a few isolated results it is essentially a non-algorithmic theory. The main tool in our paper will be Poincare Theorem and Steinhaus Chessboard Theorem. The proof of it in the case \( n = 2 \) and \( 3 \) is algorithmic, which allows to determine the above equilibrium point.

Nonseparable Complete Erdős Spaces and Submeasures on Uncountable Cardinals

Kirsten Valkenburg, Jan Dijkstra and Dave Visser

Vrije Universiteit Amsterdam, NETHERLANDS, kivalken@few.vu.nl

Among the applications of their topological characterization of complete Erdős space, \( \mathcal{E}_c \), Dijkstra and van Mill state one about copies of \( \mathcal{E}_c \) in Banach spaces \( l^p \) and one concerning Polishable \( F_\sigma \)-ideals on \( \omega \). Inspired by our previous success in lifting the first result to so-called nonseparable complete Erdős spaces in \( l^p_\kappa \) for arbitrary infinite cardinal numbers \( \kappa \), we now attempt to generalize the second. We find a partial generalization for submeasures on \( \kappa \) and related ideals. For a certain subclass, we state a full generalization, which links the ideals with nonseparable complete Erdős spaces.
Selections and Parametrizations

Vesko Valov

Nipissing University, CANADA, veskov@nipissingu.ca

We discuss applications of some selection theorems in obtaining parametric results.

Hyperspaces Homeomorphic to $\ell_2$

Rostyslav Voytsitskyy

Ivan Franko National University of Lviv, UKRAINE
voytsitski@mail.lviv.ua

In this talk we characterize metric spaces $X$ whose hyperspaces $\text{Cld}_H(X)$ and $\text{Bdd}_H(X)$ of closed and closed bounded subsets are homeomorphic to the separable Hilbert space $\ell_2$. For a metric space $(X, d)$ by $\text{Cld}_H(X)$ we denote the space of non-empty closed subsets of $X$ endowed with the topology generated by the Hausdorff “metric”

$$d_H(A, B) = \max\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(b, A)\}.$$ 

**Theorem 1.** The hyperspace $\text{Bdd}_H(X)$ (resp. $\text{Cld}_H(X)$) of a metric space $(X, d)$ is homeomorphic to $\ell_2$ if and only if $X$ is a topologically complete nowhere locally compact space and the completion $\overline{X}$ of $X$ is proper (resp. compact), connected, and locally connected.

Applying this theorem to the metric spaces $\mathbb{R} \setminus \mathbb{Q}$ and $I \setminus \mathbb{Q}$ of irrational numbers on the real line and the interval $I = [0, 1]$, we obtain the following results due to W. Kubiš and K. Sakai.

**Corollary** The hyperspaces $\text{Cld}_H(I \setminus \mathbb{Q})$ and $\text{Bdd}_H(\mathbb{R} \setminus \mathbb{Q})$ are homeomorphic to $\ell_2$.

As a by-product of the proof of Theorem 1 we obtain the following characterizations of metric spaces whose hyperspaces are separable absolute retracts.

**Theorem 2.** The hyperspace $\text{Bdd}_H(X)$ (resp. $\text{Cld}_H(X)$) of a metric space $X$ is a separable AR if and only if the completion $\overline{X}$ of $X$ is proper (resp. compact), connected and locally connected.
Criteria of Convexity for Domain of Euclidean Space

Iryna Vygovska

Institute of Mathematics, NAS of Ukraine, Tereshchenkivs’ka str., 3, Kyiv, 01601, UKRAINE, zel@imath.kiev.ua

Topological Equivalence of the Pseudoharmonic Functions Defined on the $D^2$

Iryna Yurchuk and Yevgen Polulyakh

Institute of Mathematics, NAS of Ukraine, Tereshchenkivs’ka str., 3, Kyiv, 01601, UKRAINE, iyurch@ukr.net

Let us remind that a pseudoharmonic function defined on $D^2 \subset C$ is a continuous function such that $f|_{\partial D^2}$ has a finite number of extrema and has only saddle critical points in the interior of $D^2$ (see [1]). The invariant of such functions was obtained in [2]. It is a combinatorial diagram $P(f)$ that is a finite connected graph with a strict partial order on its vertices. We list its properties:

A1) there exists a unique $\nabla$-subgraph $q(f)$ of $P(f)$ such that it is a simple cycle and every pair of its adjacent vertices is comparable;

A2) $\text{Cl}(P(f) \setminus q(f)) = \bigcup_i \Psi_i$, $\Psi_j \cap \Psi_i = \emptyset$ where $i \neq j$ and every $\Psi_i$ is a tree such that any two vertices $v'$, $v''$ of $\Psi_i$ are non-comparable;

A3) there is an embedding $\psi : P(f) \to D^2$ such that $\psi(P(f)) \subset D^2$, $\psi(q(f)) = \partial D^2$ and $\psi(P(f) \setminus q(f)) \subset \text{Int} D^2$;

A4) $D^2 \setminus \psi(P(f)) = \bigcup \theta_i$ where $\theta_j \cap \theta_i = \emptyset$, $i \neq j$, $\text{Cl}(\theta_i)$ is homeomorphic to a disk and $\partial \text{Cl}(\theta_i)$ contains one or two arcs of $\partial D^2$. 

References

Let \( f, g : D^2 \rightarrow R \) be pseudoharmonic functions. Two functions \( f \) and \( g \) are called topologically equivalent if there exist orientation preserving homeomorphisms \( h_1 : D^2 \rightarrow D^2 \) and \( h_2 : R \rightarrow R \) such that \( g = h_2 \circ f \circ h_1 \).

**Theorem 1.** Two pseudoharmonic functions \( f \) and \( g \) are topologically equivalent if and only if there exists an isomorphism \( \varphi : P(f) \rightarrow P(g) \) preserving the strict partial orders on them.

Now we will consider the inverse problem of the realization of some finite graph as a combinatorial invariant of a pseudoharmonic function.

Let \( G \) be a finite connected graph with a strict partial order on its vertices. In order to be a combinatorial diagram of some pseudoharmonic function \( G \) should comply with properties A1)–A4) and we add some extra one’s.

**Cr-cycle** of \( G \) is a subgraph \( \gamma \subset G \) such that it is the simple cycle and every its adjacent pair of vertices is comparable.

Let \( G \) satisfies following conditions:

1. **B1)** a graph \( G \) has a unique Cr-cycle \( \gamma \);

2. **B2)** if we throw away all edges which belong to \( \gamma \) from \( G \) we obtain a forest \( F = \bigcup \Psi_i \) where any \( \Psi_i \) is a tree such that its all terminal vertices belong to \( \gamma \).

A graph \( G \) which satisfies to B1)–B2) is called **\( \mathcal{D} \)-planar** if there is an embedding \( \varphi : G \rightarrow D^2 \) such that \( \varphi(\gamma) = \partial D^2, \varphi(G \setminus \gamma) \subset \text{Int} D^2 \).

There is a cyclic order on \( \gamma \) generated by an orientation of the disk. For every \( i \) this cyclic order generates cyclic order on a set \( V^*_i = \gamma \cap V(\Psi_i) \) (here \( V(\Psi_i) \) is a set of vertices of a tree \( \Psi_i \)).

Let \( T \) be a finite tree with a fixed cyclically ordered subset of vertices \( V^* \supseteq V_{\text{ter}} \) where \( V_{\text{ter}} \) is a set of terminal vertices of \( T \). A tree \( T \) is called **\( \mathcal{D} \)-planar** if there exists an embedding \( \varphi : T \rightarrow \mathbb{R}^2 \) that satisfies following conditions: \( \varphi(T) \subseteq D^2, \varphi(T) \cap \partial D^2 = \varphi(V^*) \) and if \( |V^*| \geq 3 \) then a cyclic order \( \varphi(C) \) on a set \( \varphi(V^*) \) coincides with a cyclic order generated by an orientation of \( \partial D^2 \cong S^1 \).

**Theorem 2.** If \( |V^*| = 2 \) then a tree \( T \) is \( \mathcal{D} \)-planar.

If \( |V^*| \geq 3 \) then the \( \mathcal{D} \)-planarity of a tree \( T \) is equivalent to the following condition being satisfied: for every edge \( e \) there are exactly two paths such that they pass through an edge \( e \) and connect adjacent vertices of \( V^* \).

**Theorem 3.** A graph \( G \) satisfying B1)-B2) is \( \mathcal{D} \)-planar if and only if every tree \( \Psi_i \) with a subset of vertices \( V_i^* \) that has a cyclic order defined
by $\gamma$ is $\mathcal{D}$-planar and for any indices $m \neq n$ a set $V_m^* \cap V^n_*$ belongs to a unique connected component of $\gamma \setminus V_m^*$.

Let consider any two vertices $v_1, v_2 \in V_1^*$ of some subgraph $\Psi_i$ of $G$ satisfying (B1)-(B2). The set $\gamma \setminus \{v_1, v_2\}$ consists of a disjoint union of two arcs $\gamma_1$ and $\gamma_2$.

A pair of vertices $v_1, v_2 \in V_1^*$ is called a boundary pair if either $\gamma_1$ or $\gamma_2$ does not contain any vertices of $V_2^*$ and at least one vertex of $V_1^* \setminus V_2^*$ belongs to it.

By $\omega(v_1, v_2)$ and $\alpha_i$ we denote a boundary pair and a set $\gamma_k$ from definition 3, respectively. It is clear that for every vertex $v_s$ of a boundary pair $\omega(v_1, v_2)$ there exists an adjacent vertex $\tilde{v}_s$ such that $\tilde{v}_s \in \alpha_i$ where $s \in 1, 2$.

A graph $G$ is called a $\Delta$-graph if it satisfies (B1)-(B2) and the following conditions:

B3) if for some vertices of $G$ it is true that $v < v'$ ($v > v'$) and $v', v'' \in \Psi_i \subseteq Cl(G \setminus \gamma)$ then $v < v''$ ($v > v''$);

B4) for any vertex $v \in G \setminus \gamma$ it is true that $\deg(v) = 2s \geq 4$;

B5) for any boundary pair $\omega(v_1, v_2) \in V_1^*$ a pair of adjacent vertices $\tilde{v}_1, \tilde{v}_2 \in \alpha_i$ belongs to a unique set $V_k^* \subseteq V_1^* \setminus V_2^*$;

B6) for any vertex $v$ of $\gamma$ and its adjacent vertices $v_1$ and $v_2$ such that $v_1, v_2 \in \gamma$ it holds that: if $\deg(v) = 2$ then $\deg(v_1) > 2$, $\deg(v_2) > 2$ and there exists a unique index $i$ such that $v_1, v_2 \in \Psi_i$; if $\deg(v) = 2s > 2$ ($\deg(v) = 2s + 1$) then $v_1 < v > v_2$ or $v_1 > v < v_2 (v_1 < v < v_2$ or $v_1 > v > v_2$);

B7) $G$ is $\mathcal{D}$-planar.

**Theorem 4.** If a graph is a combinatorial diagram of a pseudoharmonic function $f$ then $G$ is $\Delta$-graph.

If a graph $G$ is $\Delta$-graph then a strict partial order on $V(G)$ can be extended to one so that a graph $G$ with new partial order on a set of vertices will be isomorphic to combinatorial diagram of some pseudoharmonic function $f$. A strict partial order of a graph $G$ is the same as a strict partial order of a combinatorial diagram $P(f)$ of a pseudoharmonic function $f$ if and only if $G$ satisfies:

B8) if vertices $v', v''$ are non comparable then from $v > v'$ follows $v > v''$ where $v \in G$, $v \neq v'$, $v \neq v''$. 
Characterization Theorem for Anti-Cantor Set

Ihor Zarichnyi

Ivan Franko National University of Lviv, UKRAINE
ihar.zarichnyj@gmail.com

Let $G_i$ be sets, $|G_i| = 2$, and $0 \in G_i$ for all $i \in \mathbb{N}$. We denote

$$X = \bigoplus_{i=1}^{\infty} G_i = \{(a_1, a_2, \ldots, a_k, 0, 0, \ldots) \mid g_i \in G_i, \forall i \in \{0, \ldots, k\}, k \in \mathbb{N}\}$$

and let $d$ be a metric on $X$ defined by the formula

$$d((a_1, a_2, \ldots), (b_1, b_2, \ldots)) = \min\{i \mid a_j = b_j \text{ for all } j > i\}.$$ 

The obtained metric space is an asymptotic counterpart of the standard middle-third Cantor set and we call it the anti-Cantor set.

Metric spaces $(A, d)$ and $(B, \rho)$ are called asymptotically coarse equivalent, if there exists a multivalued map $f: A \to B$, such that, for all $\epsilon > 0$, there exist $\alpha, \beta > 0$ with the following properties: for all $x_1, x_2 \in X$, $y_1, y_2 \in Y$ we have $d(x_1, x_2) < \epsilon \Rightarrow \rho_H(f(x_1), f(x_2)) < \alpha, \rho_H(y_1, y_2) < \epsilon \Rightarrow d_H(f^{-1}(y_1), f^{-1}(y_2)) < \beta$. Here the subscript $H$ means that the Hausdorff metric is considered.

Characterization theorem for anti-Cantor set. A metric space $(Y, \rho)$ is asymptotically coarse equivalent to the anti-Cantor set if and only if there exist $a > 0$, $n \in \mathbb{N}$, there exist strongly increasing sequences $(a_i)_{i \in \mathbb{N}}$ and $(n_i)_{i \in \mathbb{N}}$ of natural and real numbers respectively that tend to infinity such that for every $i$ the set $Y_i$ can be represented as disjoint union of a countable family of sets $\{Y_j\}_{j \in \mathbb{N}}$ such that, for all $j, k \in \mathbb{N}$, diam$(Y_j) \leq a_i$, dist$(Y_j, Y_k) > a_{i-1}$, and every set $Y$ can be covered by at least $2^{n_i-n}$ and at most $2^{n_i}$ sets of diameter $\leq a$. 

References


Spaces of Idempotent Measures: a Survey of Results

Mykhailo Zarichnyi

Ivan Franko National University of Lviv, 1 Universytetska Str., 79000 Lviv, UKRAINE, mzar@network.lviv.ua

The idempotent mathematics is a part of mathematics dealing with some idempotent operations (e.g., \( \oplus = \max \)) instead of usual addition and multiplication. In the recent decade, the results and methods of the idempotent mathematics found numerous applications not only in different parts of (traditional) mathematics but also in dynamical optimization, economics, physics etc [3].

We consider the spaces of the so called idempotent measures (Maslov measures) on compact Hausdorff spaces endowed with the weak topology. A measure \( \mu \) is a Maslov measure if \( \mu(A \cup B) = \max\{\mu(A), \mu(B)\} \) for all sets \( A, B \). We identify the set of Maslov measures with the set of all functionals \( \mu: C(X) \to \mathbb{R} \) satisfying \( \mu(c) = c, \mu(\varphi + c) = \mu(\varphi) + c, \) and \( \mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi) \).

The construction of the space of idempotent measures determines a functor on the category of compact Hausdorff spaces that shares some topological and categorical properties with the functor of probability measures. This functor is normal in the sense of E. Shchepin [4]. In particular, we show that, for this functor, a counterpart of the Kolmogorov theorem holds. We also show that the functor of idempotent measures preserves the class of open maps. The corresponding result for the probability measures is proved by Ditor and Eifler [1]. We also prove the existence of a counterpart of the Milyutin map for the spaces of idempotent measures.

Some geometric properties of the space of idempotent measures are considered. In particular, we prove that the space of idempotent measures of any infinite compact metrizable space is homeomorphic to the Hilbert cube. We also discuss the properties of the spaces of idempotent probability measures for nonmetrizable compacta.

It is known that the correspondence assigning to every probability measures the set of measures with given marginals on the product
is continuous with respect to the Vietoris topology [2]. We demonstrate that this is not the case for the correspondence of idempotent measures on the product.

References

Complete Nonmeasurable Unions

Szmyon Żeberski and Robert Ralowski

Institute of Mathematics and Computer Science, Wroclaw University of Technology, POLAND, szymon.zeberski@pwr.wroc.pl

Assume that $2^\omega$ is the minimal quasi-measurable cardinal i.e. there exists $\sigma$-additive ideal $I$ of subsets of $2^\omega$ such that the Boolean algebra $P(2^\omega)/I$ satisfies c.c.c. We show that for a c.c.c. $\sigma$-ideal $\mathcal{I}$ with a Borel base of subsets of an uncountable Polish space, if $\mathcal{A}$ is a point-finite family of subsets from $\mathcal{I}$ then there is a subfamily of $\mathcal{A}$ whose union is completely nonmeasurable i.e. its intersection with every non-small Borel set does not belong to the $\sigma$-field generated by Borel sets and the ideal $\mathcal{I}$. This result is a generalization of Four Poles Theorem (see [1]) and result from [2]. The same statement was shown in [3] under the assumption that there is no quasi-measurable cardinal not greater than $2^\omega$.

References
Some Unsolved Problems of Complex Convex Analysis

Yuri Zelinskii

Departement of Complex Analysis, Institute of Mathematics, NAS of Ukraine, Tvershenkivs'ka str., 3, Kyiv, 01601, UKRAINE, zel@imath.kiev.ua
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Appendix

$G_δ$-Points in Compact Spaces

Henryk Michalewski, Adam Krawczyk
and Witold Marciszewski

Institute of Mathematics, Warsaw University, POLAND
henrykmichalewski@gmail.com

For a compact space $K$ we consider the set $FC(K)$ of all $G_δ$-points in $K$, i.e., points with countable base of neighborhoods in $K$. We show that, for every scattered Eberlein compact space $K$, the set $FC(K)$ is a $G_δ$ set in $K$. We also give an example of a scattered Eberlein compactum with non-metrizable set $FC(K)$. Moreover, we give an example of a Corson compact space $K$ such that $FC(K)$ does not contain any dense $G_δ$ subset of $K$. This answers three questions of Tkachuk.
The Topological Classification of $m$-Functions on Noncompact Surfaces of Finite Type

Kateryna Mishchenko

National Aviation University, Kyiv, UKRAINE, mieschenko.katya@gmail.com

In this lecture we shall completely classify noncompact surfaces with an arbitrary number of component of the boundary. A first attempt to classify noncompact surfaces appeared in Kerényi’s “Vorlesungen über Topologie” in 1923. Later this theme was considered by Richards. In this lecture we present a complete topological classification of noncompact surfaces with any number of the boundary components. The main objects of our investigation are triangulable connected surfaces. To handle noncompact surfaces we introduce several new definitions and invariants: a boundary component (or end) and the ideal boundary (or the set of ends) of a non-compact surface. Then, defining four “orientability classes” of surfaces and genus, we describe characteristic properties of ideal boundaries of those noncompact surfaces with any number of boundary components. Thus, a task is to classify surfaces without boundary.

Regular and Critical Points of Continuous Functions

Yevgen Polulyakh

Institute of Mathematics, NAS of Ukraine, Tereshchenkivs'ka str., 3, Kyiv, 01601, UKRAINE, polulyah@imath.kiev.ua